Endogenous Production Networks and Firm Dynamics

Anuraag Aekka
NYU Stern

Gaurav Khanna
UCSD

August 2024

Updated regularly, please click here for latest version.

Abstract

This paper studies the role of customer and supplier acquisition in shaping firm dynamics and aggregate productivity. Using transaction-level data from a large Indian state, we document lifecycle patterns of customer and supplier networks. We find that younger firms have fewer customers and suppliers, lower sales and intermediate expenditures, and higher output prices and input costs. Motivated by these patterns, we develop a model of endogenous network formation where heterogenous firms undertake costly acquisition of customers and suppliers over the lifecycle. We study the normative properties of the model and find that the decentralized equilibrium is inefficient due to vertical and search externalities. Inefficient pricing and acquisition choices lead to quantitatively large aggregate productivity losses. We use the model to study how differences in acquisition technology map to productivity differences. We find that improvements in acquisition technology can generate sizable productivity gains, and that improvements in allocative efficiency are central for delivering these gains.

JEL Codes: D24, D61, D62, E22, F14, L14

We are highly grateful to Venky Venkateswaran and Michael Waugh for their generous advice and support. We also thank Jess Benhabib, Corina Boar, Giulia Brancaccio, Francisco Buera, Gian Luca Clementi, Niklas Engbom, Simon Gilchrist, Boyan Jovanavic, Ricardo Lagos, Guido Menzio, Virgiliu Midrigan, Alessandra Peter, Paul Scott and seminar participants at NYU, Washington University in St. Louis, San Francisco Fed, and BSE Summer Forum for valuable comments.

1 Introduction

Recent empirical work suggests that firms grow, to a large degree, through matching with trading partners (Afrouzi et al. (2023), Einav et al. (2022), Argente et al. (2023)). When firms acquire new customers or suppliers, or separate from existing ones, they create and destroy links in the production network. Thus the acquisition choices of firms have important implications for the structure of the production network. Furthermore, when there gains from variety or complementarities in production, the structure of the production network matters for aggregate productivity. However, thus far we have limited understanding of how acquisition efforts of firms shape aggregate productivity, mainly due to the difficulty of modeling dynamic customer and supplier networks in an endogenous production network setting. Moreover, differences in acquisition efforts are potentially important drivers of productivity differences, as the technologies firms have to match with one another appear to vary across country and over time, for example, due to differences in information technology (Jensen (2007), Aker (2010), Goyal (2010)) or legal institutions (Boehm and Oberfield (2020), Boehm (2022)). The objective of this paper is to develop a model of endogenous network formation to study the implications of customer and supplier acquisition for the production network and aggregate productivity.

We make four contributions. First, using unique firm-to-firm data from a large Indian state[1], we document lifecycle patterns of customer and supplier networks which support a theory of firms growing through customer and supplier acquisition. Second, motivated by these patterns, we develop a model of endogenous network formation where heterogeneous firms undertake costly acquisition of customers and suppliers over the lifecycle. Our model maintains tractability, despite the introduction of dynamic customer and supplier networks. Third, we study the normative properties of this environment and find quantitatively large aggregate productivity losses from inefficient pricing and acquisition choices. Calibrating the model to the Indian data, we find that aggregate productivity in the efficient allocation is 16% greater than in the decentralized equilibrium. Fourth, we use the model to study how differences in acquisition technology map to productivity differences. We find that improvements in acquisition technology can generate sizable productivity gains. In a counterfactual exercise, using a moment from Arkolakis et al. (2023) which suggests Chilean firms are able to scale their customer and supplier networks more easily than Indian firms, we find that aggregate productivity in India would be 3% greater if Indian firms could scale their trading partners as easily as Chilean firms. Improvements in allocative efficiency are central for delivering these gains.

[1] The state is twice Chile's population and three times Belgium's, both popular sources of similar data.

In the first part of our paper, we document lifecycle patterns of customer and supplier networks using unique firm-to-firm data from a large Indian state. The data cover the near universe of transactions, where at least one node of the transaction lies within the state. We find that younger firms have fewer customers and suppliers, and lower sales and intermediate expenditure. We then view the data through the lens of a CES demand system. This allows us to map trade flows between firms to the input costs firms face and the output prices they charge. We find that younger firms face higher input costs and charge higher output prices. These patterns support a theory of firms growing through acquiring customers and suppliers.

In the second part of our paper, we develop a novel model of endogenous network formation where firms undertake costly acquisition of trading partners over the lifecycle. In the model, there exists a continuum of monopolistically competitive firms which produce unique varieties using labor and varieties produced by other firms. The economy features search frictions, which entail that a firm is only able to trade with partners it is connected to. In order to connect with trading partners, in each period, firms exert costly acquisition effort, both for matching with customers and suppliers, and matches are made via an aggregate matching function. Importantly, the cost of acquisition effort is strictly convex, creating an incentive for firms to spread out acquisition efforts over time. At the firm level, these acquisition efforts give rise to the lifecycle patterns observed in the data. At the aggregate level, these acquisition efforts give rise to an endogenous production network. This search and matching process is what we refer to as the "acquisition technology".

In general, introducing dynamic customer and supplier networks into an endogenous production network model adds significant complexity. When firms decide how much effort to exert in acquiring partners, they have to consider how potential partners will evolve over time. However, how a given customer (supplier) evolves over time depends itself on how the customer's (supplier's) own customers and suppliers evolve over time. This logic continues ad infinitum, such that an individual customer's (supplier's) evolution depends on the evolution of all firms upstream and downstream of it. In our setting, we maintain tractability by making assumptions on the acquisition technology and firm productivity process which imply that, in equilibrium, the age and productivity of a partner become a sufficient type to describe it.

We study the normative properties of this environment. We find that the decentralized equilibrium is inefficient due to vertical and search externalities. Firms set prices too high and underutilize intermediate inputs relative to the efficient allocation due to double marginalization. In addition, firms make inefficient acquisition choices due to the presence of vertical and congestion externalities. When choosing acquisition effort, firms do not internalize the surplus they generate for partners they match with. In addition, firms fail to internalize how their customer (supplier) acquisition effort reduces matching probabilities for other firms searching for customers (suppliers). We find that inefficient pricing and acquisition choices generate large aggregate losses. Calibrating the model to the Indian data, we find that aggregate productivity in the efficient allocation is 16% greater than in the decentralized equilibrium.

Finally, we use the model to study how differences in acquisition technology map to productivity differences. In the model, this technology is summarized by two parameters, capturing the level and curvature of acquisition costs. When firms are able to scale their trading partners more easily, due to lower curvature in acquisition costs, high-productivity firms expand acquisition relative to low-productivity firms. As a result, the production network features a greater share of links between high-productivity firms. This greater concentration of links between high-productivity types generates higher aggregate productivity, even when the total number of links is the same. On the other hand, when the level of acquisition costs is lower, all firms acquire more partners. The resulting network features more firm-to-firm links, generating higher aggregate productivity as firms benefit from gains from variety.

To quantify the effects of these channels on aggregates, we calibrate the model to match key features of the data. In particular, the curvature of acquisition costs is disciplined by the elasticity between number of customers and sales. In our data, we estimate this elasticity to be 0.36. For comparison, Arkolakis et al. (2023) find a somewhat higher elasticity in Chilean firm-to-firm data, suggesting Chilean firms are able to scale their customer and supplier networks more easily. In a counterfactual exercise, we recalibrate the model to target the Chilean moment, holding fixed the total number of connections. We find that aggregate productivity increases by 3% relative to the baseline. As the total number of connections is held fixed, this productivity gain is due to the production network being more concentrated in links between high-productivity firms. Furthermore, improvement in allocative efficiency plays a central role in delivering this productivity gain. In inefficient economies, changes in technology affect aggregate productivity through affecting both technical efficiency, i.e. productivity in the efficient allocation, and allocative efficiency, i.e. distance of the decentralized allocation from the efficient allocation. We find that roughly 4/5 of the 3% gain in aggregate productivity is due to improvement in allocative efficiency, while 1/5 is due to improvement in technical efficiency.

As for the level of acquisition costs, the elasticity of aggregate output with respect to the level of acquisition costs depends only on the intermediate share, the elasticity of substitution across inputs, and the curvature of acquisition costs. Under our calibration, a 10% reduction in the level of acquisition costs leads to a 1.0% increase in aggregate productivity.

This paper contributes to several strands of the literature studying endogenous production networks. This literature has taken seriously the idea that the production network is an endogenous object arising out of individual firm decisions and has documented numerous facts on firm-to-firm trade. Empirically, we contribute to this literature by adding facts on the lifecycle dimension of trade.

Theoretically, we relate to models in this literature in which trading partners evolve over time. Lim (2018) studies a model in which the value of relationships vary over time due to idiosyncratic shocks. In their model, dynamics of customers and suppliers arise out of the dynamics of idiosyncratic shocks. Huneeus (2020) builds on this framework by introducing adjustment costs which prevent firms from readjusting customers and suppliers. Boehm et al. (2024) study a model in which potential suppliers arrive randomly according to a Poisson process. We differ from existing work by modeling dynamics which arise from a random search process. In our environment, dynamics arise out of firms being unable to match immediately with their steady-state set of partners due to the presence of search frictions. Instead, firms slowly acquire customers and suppliers over time through exerting costly acquisition effort. This allows us to generate the lifecycle patterns we document in the data. Our contribution is to demonstrate that inefficiency in customer and supplier acquisition (search effort) can lead to quantitatively large aggregate productivity losses, and that changes in allocative efficiency are central for understanding how technology differences map to productivity differences.

In modeling the production network as being formed through search and matching, we relate to Arkolakis et al. (2023) and Demir et al. (2023). We extend such models by introducing dynamics in customer and supplier networks.

Incorporating dynamics which arise from search frictions, however, introduces significant complexity. When deciding how much effort to exert in acquiring partners, firms require conjectures about how partners' payoff-relevant attributes will evolve over time (e.g. the intermediate demands of customers, and the output prices of suppliers). The evolution of these payoff-relevant attributes depends on the acquisition choices of the partner, but also on the evolution of their own partners' attributes. Thus, in equilibrium, a firm's conjecture about how a given partner's attributes evolve has to be consistent with not only the acquisition choices of the partner, but also the acquisition choices of all firms upstream and downstream of the partner.

In our setting, we maintain tractability by making assumptions on the acquisition technology and firm productivity process which imply that, in a stationary equilibrium, the age and productivity of a partner become a sufficient state to describe it. As a result, firms can instead use conjectures about the attributes of given states, and understand how partners transition through the state space over time.

The efficiency properties of our model are similar to those of several endogenous production network models. Though we differ by featuring dynamic customer and supplier networks, the underlying inefficiencies in our model are the same as those in models which rely on search and matching as the network formation technology: Arkolakis et al. (2023), Demir et al. (2023). The inefficiencies in our model are also related to those in endogenous network models which rely on other network formation technologies (Lim (2018), Huneeus (2020), among others), as even in these models, firms do not fully internalize how their network formation choices affect their partners. Again, our contribution is to demonstrate that aggregate losses from inefficiencies can be quantitatively large, and that changes in allocative efficiency play a central role in understanding how technology differences map to productivity differences.

This paper also contributes to the literature on customer capital. Theoretically, this literature has posited that firms must spend resources to match with trading partners due to the presence of search frictions (e.g. Drozd and Nosal (2012), Gourio and Rudanko (2014), Arkolakis (2010)). This is also the case in our model. However we differ from the customer capital literature by embedding firms in an endogenous network. As a result, acquisition efforts have implications for upstream and downstream firms and the structure of the production network. Empirically, this literature has documented the importance of customer growth in explaining sales growth. Examples of recent work include Afrouzi et al. (2023), Argente et al. (2023), Einav et al. (2022), and Fitzgerald et al. (2023). Our lifecycle patterns align with the findings of this literature, that firms grow through expanding their customer networks. We also add complementary patterns on the supplier side.

The paper is organized in the following way. In Section 2, we describe our data and document lifecycle facts on customers and suppliers. In Section 3, we describe our model. In Section 4, we describe our estimation procedure and discuss firm dynamics in our setting. In Section 5, we study the normative properties of our model. In Section 6, we study how the acquisition technology shapes aggregate productivity.

2 Data and Lifecycle Facts

2.1 Data

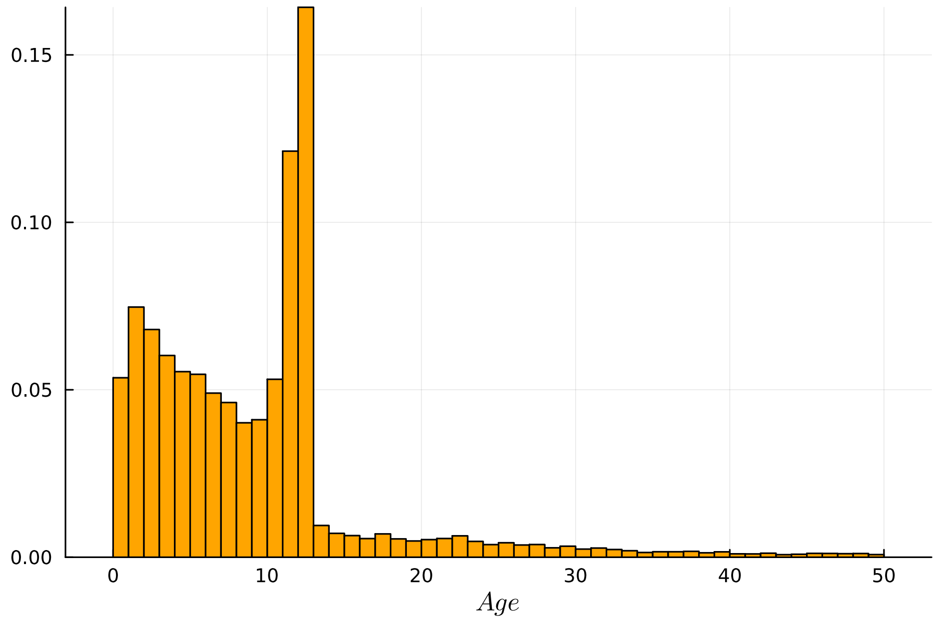
Data on the production network comes from daily transactions between registered establishments in a large Indian state and all registered establishments throughout India or abroad. In April 2018, the state tax authority created an E-Way Bill System to improve tax compliance. Under the new system, any transaction with value exceeding 50,000 Rs (700 USD) must be reported electronically using the system. The system generates a waybill which the transporter must carry during shipment. The waybill contains the Permanent Account Number (Tax ID) of the supplier and customer, the 4-digit HSN code of the product, and the value of the shipment. We define a firm to be the combination of a Tax ID and HSN.

We add data on firm age from two separate sources. The first source is state records on various types of registration (e.g. year of incorporation, year of registration with state tax authority, etc.). We call the first year a firm is registered with the state as the registration year of the firm. The state registration data provides registration years for 1,100,000 firms in our sample. The second source is a large online platform connecting buyers and sellers called IndiaMART. IndiaMART is the largest online B2B marketplace in India and contains rich information about the firms on the platform. Included in this information is the year of establishment. IndiaMART provides a year of establishment for 75,000 firms in our sample. We assign the birth year of a firm as the minimum between the year of establishment and the year of registration. In total, we can assign birth years for 1,170,000 firms in our sample. We assign the firm's age as the difference between the year 2018 and the firm's birth year.

We use the sample of transactions which occur between April 2018 and March 2019. We drop any firms which are born after 2017 in order to ensure we observe a full year of transactions for every firm in the sample. Lastly, as will be explained further below, we also only keep the largest connected set of firms within each HSN. This leaves us with 2,900,000 links between 1,700,000 firms.

In Figure 1, we plot the age distribution of firms. Specifically, we plot the fraction of Tax IDs with a given age. As can be seen in the figure, there is a large mass of Tax IDs with ages between 11 - 13. This is due to the introduction of a new state value added tax system in the year 2005. Under the new law, many existing firms registered with the state authority. As this makes it difficult to determine the true age of these firms, in our empirical work, we group firms 11 and older into a single category.

Figure 1: Distribution of Firm Age



Note: We plot the fraction of Tax IDs with a given age. The large mass of firms between ages 11-13 is due to the introduction of a new VAT system in 2005, under which many existing firms registered with the state.

2.2 Lifecycle Facts on Customers and Suppliers

In this section, we document lifecycle patterns of customer and supplier networks. Younger firms have fewer customers and suppliers, lower sales and intermediate expenditures, and higher output prices and input costs. The patterns support a theory of firms growing through acquisition of customers and suppliers over time. Motivated by these patterns, in Section 3, we develop an endogenous network model which features customer and supplier acquisition.

We first document patterns which do not rely on a demand system. We call these patterns "Facts". We then view the data through the lens of a CES demand system to map trade flows between firms to the input costs firms face and the output prices they charge. We call these findings which require the assumption of a CES demand system as "CES Facts". For all of the patterns we document, we estimate fixed effects for 4 age categories: Age 1-3, Age 4-6, Age 7-10, Age 11+. We refer to firms in the Age 1-3 category as "entrants".

Fact 1 *Younger firms have fewer customers and lesser sales.*

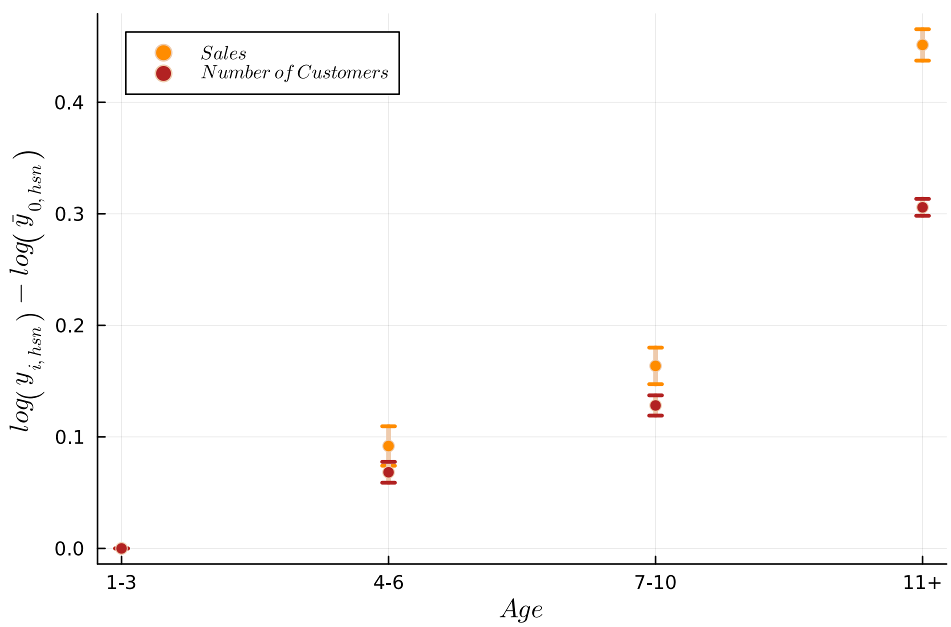
Figure 2 displays lifecycle patterns of number of customers and sales within an HSN. In particular, for each variable of interest y, we estimate the following regression equation:

$$log\left(y_{i,hsn}\right)=\sum_{a}\gamma_{a}\mathbf{1}(age_{i}\in a)+\sum_{h}\gamma_{h}\mathbf{1}(hsn=h)+\varepsilon_{i,hsn} \tag{1}$$

Here, hsn refers to the 4-digit HSN within which the firm sells, i refers to the firm's Tax ID, $\mathbf{1}(age_i \in a)$ is an indicator variable which equals 1 if the age assigned to the Tax ID is in age category a, and $\mathbf{1}(hsn = h)$ is an indicator variable which equals 1 if the firms sells its product in the 4-digit HSN category h.

In Figure 2, we display the estimates of interest, γ_a, normalizing by youngest age group. The normalized estimates express the difference in log number of customers and log sales for firms in a given age category relative to entrants who sell products in the same HSN. Firms which are 11+ have 36% more customers within an HSN than entrants in the same HSN. Firms which are 11+ have 57% greater sales within an HSN than entrants in the same HSN.

Figure 2: Sales and Number of Customers



Note: We plot estimated age fixed effects, γ_a, from Equation 1 along with bootstrap standard errors, normalizing by the youngest age group. The normalized estimates express the difference in log number of customers (within an HSN) and log sales (within an HSN) for firms in a given age category relative to entrants who sell products in the same HSN.

Fact 2 *Younger firms have fewer suppliers and lesser intermediate expenditure.*

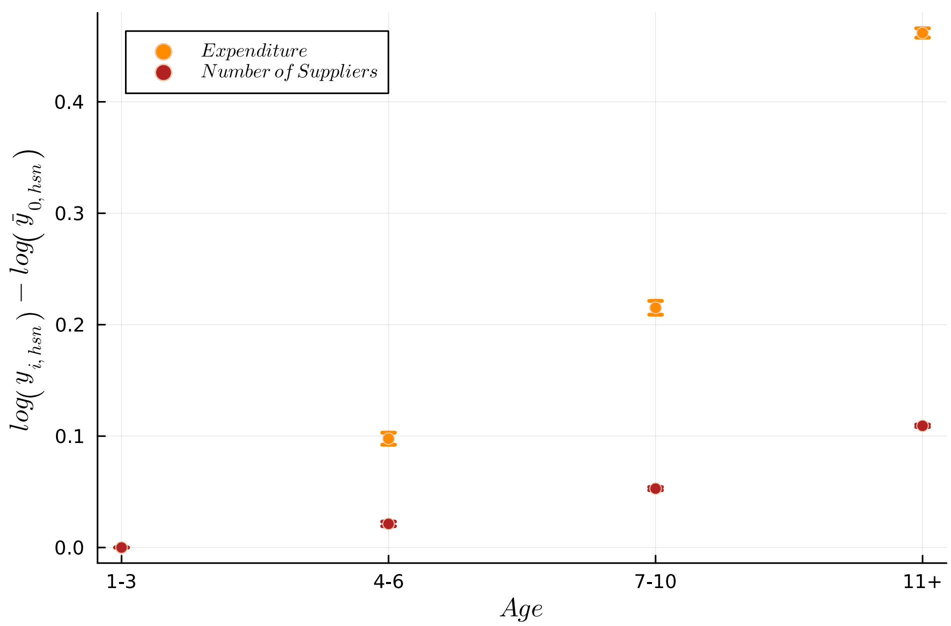
Figure 3 displays lifecycle patterns of number of suppliers and intermediate expenditure within an HSN. In particular, for each variable of interest y, we estimate the following regression equation:

$$log\left(y_{i,hsn}\right) = \sum_{a} \gamma_a \mathbf{1}(age_i \in a) + \sum_{h} \gamma_h \mathbf{1}(hsn = h) + \varepsilon_{i,hsn} \tag{2}$$

Here, hsn refers to the 4-digit HSN from which the firm purchases its inputs, i refers to the firm's Tax ID, $\mathbf{1}(age_i \in a)$ is an indicator variable which equals 1 if the age assigned to the Tax ID is in age category a, and $\mathbf{1}(hsn = h)$ is an indicator variable which equals 1 if the firms purchases its inputs from from the 4-digit HSN category h.

In Figure 3, we display the estimates of interest, γ_a, normalizing by youngest age group. The normalized estimates express the difference in log number of suppliers and log intermediate expenditure for firms in a given age category relative to entrants who purchase inputs from the same HSN. Firms which are 11+ have 11% more suppliers in an HSN than entrants who purchase inputs from the same HSN. Firms which are 11+ have 59% greater intermediate expenditure in an HSN than entrants who purchase inputs from the same HSN.

Figure 3: Intermediate Expenditure and Number of Suppliers



Note: We plot estimated age fixed effects, γ_a, from Equation 2 along with bootstrap standard errors, normalizing by the youngest age group. The normalized estimates express the difference in log number of suppliers (within an HSN) and log intermediate expenditure (within an HSN) for firms in a given age category relative to entrants who purchase inputs from the same HSN.

The positive relationship between firm size and firm age is well documented. Existing work has documented this relationship using sales or employment as the measure of size. Here, we find the same relationship using number of trading partners as the measure of size.

We now impose the assumption of a CES demand system. Viewing the data through the lens of a CES demand system allows us to map trade flows between firms to input costs firms face and output prices they charge. Specifically, we assume that a customer minimizes the cost of sourcing an HSN intermediate good, which is a CES aggregate of varieties the customer purchases from different suppliers.

$$\min_{\nu_{ij,hsn}} \sum_{i \in G_{j,hsn}} p_{i,hsn}\nu_{ij,hsn} \qquad \text{s.t.} \qquad \left(\sum_{i \in G_{j,hsn}} (q_{i,hsn}\nu_{ij,hsn})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \geq x_{j,hsn}$$

Here $x_{j,shn}$ denotes the quantity of the HSN hsn good customer j requires, $G_{j,hsn}$ denotes the set of HSN hsn suppliers customer j is connected to, and $\nu_{ij,hsn}$ denotes the quantity customer j demands from supplier i. Suppliers may vary in the quality of their variety, $q_{i,hsn}$, and charge price $p_{i,hsn}$. The elasticity of substitution across varieties is given by $\sigma > 1$. Solving the problem of the customer, the log share of HSN hsn expenditure customer j spends on supplier i (i.e. the log input share) is given by:

$$e_{ij,hsn} = (1-\sigma)log\left(\frac{p_{i,hsn}}{q_{i,hsn}}\right) - (1-\sigma)log\left(c_{j,hsn}\right)$$

The input share of customer j on supplier i within HSN hsn, $e_{ij,hsn}$, is a log-linear function of the supplier's quality-adjusted output price $\frac{p_{i,hsn}}{q_{i,hsn}}$ and the customer's quality-adjusted input cost $c_{j,hsn} = \left(\sum_{i \in G_{j,hsn}} \left(\frac{p_{i,hsn}}{q_{i,hsn}}\right)^{1-\sigma}\right)^{1/(1-\sigma)}$.

Guided by this relationship, we estimate the following regression equation:

$$e_{ij,hsn} = \psi_{i,hsn} + \phi_{j,hsn} + \varepsilon_{ij,hsn} \tag{3}$$

where i denotes the Tax ID of the supplier, j denotes the Tax ID of the customer, and hsn denotes the HSN of the expenditure. That is, we project log input share onto a supplier fixed effect, $\psi_{i,hsn}$, and a customer fixed effect $\phi_{j,hsn}$. Notice under our assumptions, $\psi_{i,hsn}$ corresponds to the supplier's quality-adjusted output price and $\phi_{j,hsn}$ corresponds to the customer's quality-adjusted input cost. We estimate Equation 3 and discuss the results below.[2]

[2]Estimating Equation 3 using OLS poses a threat to identification. To obtain unbiased estimates, the assignment of suppliers to customers must be exogenous with respect to $\varepsilon_{i,j,hsn}$, an assumption referred to as "exogenous mobility" in the labor literature (Abowd et al. (1999)). In Appendix A.1 we argue that in the case this assumption is violated, our estimated differences between young and old firms represent lower bounds. That is, the true differences in input costs and output prices between young and old firms are larger.

CES Fact 1 *Younger firms have higher input costs*

Figure 5 displays lifecycle patterns of $\phi_{j,hsn}$ from Equation 3. In particular, we estimate the following regression equation:

$$\phi_{j,hsn} = \sum_a \gamma_a \mathbf{1}(age_j \in a) + \sum_h \gamma_h \mathbf{1}(hsn = h) + \varepsilon_{j,hsn} \tag{4}$$

Here, hsn refers to the 4-digit HSN from which the firm purchases its inputs, j refers to the firm's Tax ID, $\mathbf{1}(age_j \in a)$ is an indicator variable which equals 1 if the age assigned to the Tax ID is in age category a, and $\mathbf{1}(hsn = h)$ is an indicator variable which equals 1 if the firms purchases its inputs from from the 4-digit HSN category h.

In Figure 5, we display the estimates of interest, γ_a, normalizing by the youngest age group. The normalized estimates express the difference in $\phi_{j,hsn}$ for a given age group relative to entrants who purchase inputs in the same HSN. Firms which are 11+ have fixed effects which are 0.28 less than entrants who purchase inputs from the same HSN. Noting that $\phi_{j,hsn} = (\sigma - 1) log\left(c_{j,hsn}\right)$ under our assumptions on input demand, if $\sigma = 4.30$, this implies that firms which are 11+ have 8% lower input costs than entrants.

CES Fact 2 *Younger firms charge higher output prices*

Figure 6 plots lifecycle patterns of $\psi_{i,hsn}$ from Equation 3. In particular, we estimate the following regression equation:

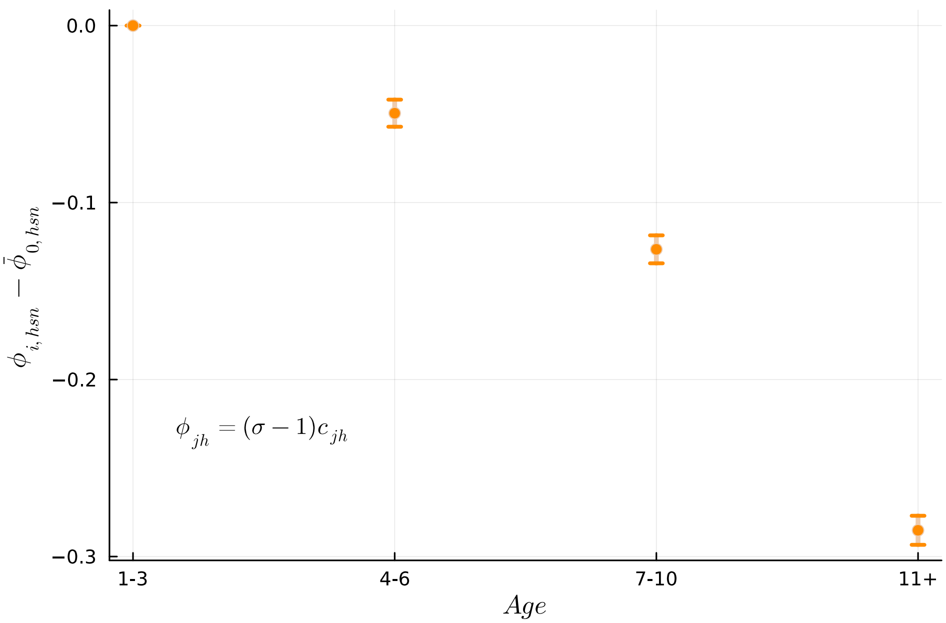
$$\psi_{i,hsn} = \sum_a \gamma_a \mathbf{1}(age_i \in a) + \sum_h \gamma_h \mathbf{1}(hsn = h) + \varepsilon_{i,hsn} \tag{5}$$

Here, hsn refers to the 4-digit HSN within which the firm sells, i refers to the firm's Tax ID, $\mathbf{1}(age_i \in a)$ is an indicator variable which equals 1 if the age assigned to the Tax ID is in age category a, and $\mathbf{1}(hsn = h)$ is an indicator variable which equals 1 if the firms sells its product in the 4-digit HSN category h.

In Figure 6, we display the estimates of interest ,γ_a, normalizing by youngest age group. The normalized estimates express the difference in $\psi_{i,hsn}$ for a given age category relative to entrants who sell products in the same HSN. Firms which are 11+ have fixed effects which are 0.07 greater than entrants in the same HSN. Noting that $\psi_{i,hsn} = (1 - \sigma) log\left(\frac{p_{i,hsn}}{q_{i,hsn}}\right)$

In order for $\psi_{i,hsn}$ to be identified, a firm must have at least 2 customers. Similarly, in order for $\phi_{j,hsn}$ to be identified, a firm must have at least 2 suppliers. Thus, we will only be able to recover fixed effects for firms in our sample which meet this criteria. Furthermore, the estimated fixed effects can only be compared within a connected set. Thus, for each HSN, we isolate the giant component (the largest connected set), and estimate Equation 3 on this set.

Figure 4: Input Share Customer Effect $\phi_{j,hsn}$



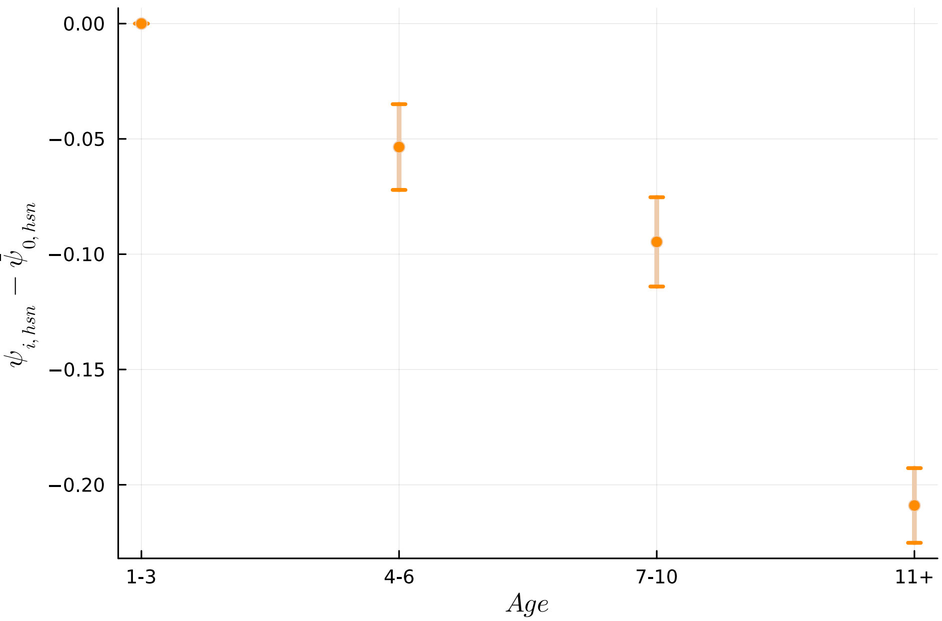
Note: We plot estimated age fixed effects, γ_a, from Equation 4 along with bootstrap standard errors, normalizing by the youngest age group. The normalized estimates express the difference in $\phi_{j,hsn}$ for firms in a given age group relative to entrants who purchase inputs in the same HSN. Under our assumptions on input demand, $\phi_{j,hsn} = (\sigma - 1)log\,(c_{j,hsn})$.

under our assumptions on input demand, if $\sigma = 4.30$, this implies that firms which are 11+ charge 2% lower output prices than entrants.

Our CES Facts rely on assumptions that a given supplier charges the same quality-adjusted price to all of its customers and that customers have a constant elasticity of substitution across suppliers. The waybills which comprise our transaction data also have an entry for quantity of good shipped. This information is not required by the tax authority, so it is frequently missing in the waybills. However for the subset of transactions for which we can observe both transaction values and quantities, we can construct unit values. In Appendix A.2, we decompose variation in these unit values. We find that variation within supplier across customers plays a small role in explaining the total variation in unit values. This lends support to our assumptions.

The lifecycle patterns we document are in line with a theory of firm dynamics in which firms slowly grow over time due to frictions in firm-to-firm matching. However, these patterns could also arise in a model in which the customer and supplier networks of a firm are determined period-by-period by the firm's idiosyncratic productivity, combined with a positive correlation between age and idiosyncratic productivity (e.g. due to survivorship

Figure 5: Input Share Customer Effect $\phi_{j,hsn}$ controlling for Sales

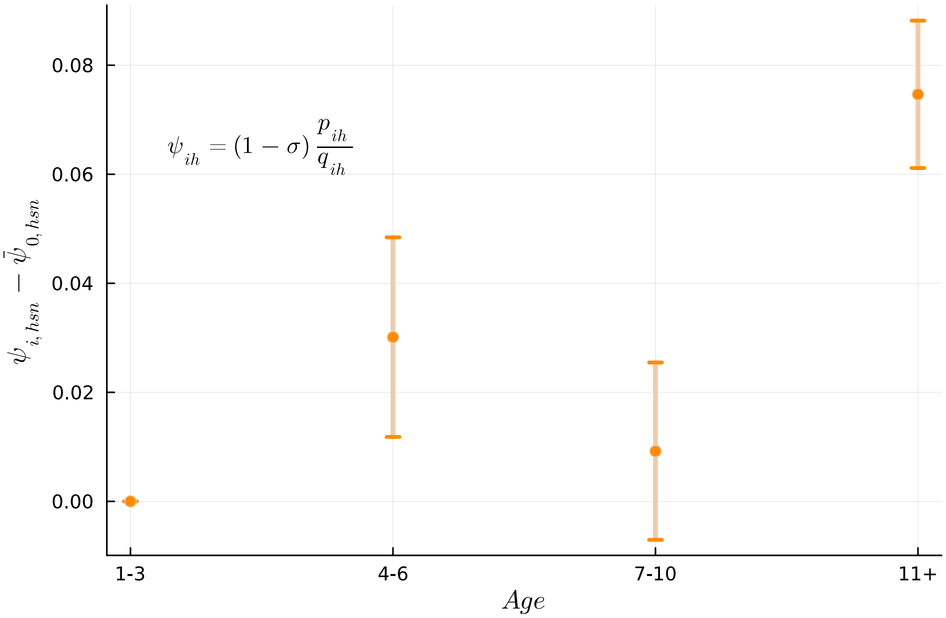


Note: We plot estimated age fixed effects, γ_a, from Equation 4 along with bootstrap standard errors, normalizing by the youngest age group. The normalized estimates express the difference in $\phi_{j,hsn}$ for firms in a given age group relative to entrants who purchase inputs in the same HSN. Under our assumptions on input demand, $\phi_{j,hsn} = (\sigma - 1)log\,(c_{j,hsn})$.

bias or "learning-by-doing"). In this case, the lifecycle patterns just reflect a positive correlation between age and idiosyncratic productivity, and matching frictions do not play an important role in shaping firm dynamics. In Appendix A.3, we repeat the empirical analysis of this section, controlling for firm sales. We find that, controlling for sales, younger firms have fewer customers and lower output prices. Furthermore, controlling for sales, younger firms have fewer suppliers, higher input costs, and lesser intermediate expenditure. These patterns suggest that frictions in firm-to-firm matching play an important role in shaping firm dynamics.

Given that we can construct unit values for a subset of transactions, an alternative we could have followed is to document lifecycle patterns in unit values. The issue with this approach, however, is that we are not able to control for quality heterogeneity as we do not observe any attributes of the products. Thus it may be the case that a firm which faces a higher unit cost, actually faces a lower cost per *effective unit* due to being provided greater quality. Similarly, it may be the case that a firm which charges a lower unit price, actually charges a higher price per *effective unit* due to providing lesser quality. Instead, in our approach, we are able to infer *price per effective unit* and *cost per effective unit* using the observed demand of customers. The assumptions we make provide a way to control for

Figure 6: Input Share Supplier Effect $\psi_{i,hsn}$



Note: We plot estimated age fixed effects, γ_a, from Equation 5 along with bootstrap standard errors, normalizing by the youngest age group. The normalized estimates express the difference in $\psi_{i,hsn}$ for firms in a given age group relative to entrants who sell products in the same HSN. Under our assumptions on input demand, $\psi_{i,hsn} = (1-\sigma)log\left(\frac{p_{i,hsn}}{q_{i,hsn}}\right)$.

quality heterogeneity, despite lacking information about product attributes.

In summary, we find that younger firms have fewer customers and suppliers, lower sales and intermediate expenditures, and higher input costs and output prices. These findings are in line with a theory in which firms grow through acquiring customers and suppliers. Acquiring customers shifts out a firm's demand, increasing sales through an extensive margin. Acquiring suppliers reduces a firm's marginal cost, allowing it to reduce its output price and sell more to existing customers. Motivated by these patterns, we develop an endogenous production network model which features customer and supplier acquisition.

3 Model of Network Formation

In this section, we develop a model of the production network in which the network is an endogenous object arising out of acquisition efforts of firms. Time is discrete. The economy is inhabited by a continuum of monopolistically competitive firms which produce unique varieties. Each firm is endowed with a permanent productivity and operates a constant returns to scale technology which uses labor and varieties produced by other firms as inputs.

The economy features search frictions which entail that a firm is only able to trade with the subset of all firms it is connected to. In order to connect with trading partners, firms, in each period, exert costly acquisition effort and match with new customers and suppliers via random search. We assume that the cost of acquisition is strictly convex. This generates the feature that firms slowly acquire customers and suppliers over the lifecycle rather than jumping immediately to a steady-state. This search and matching process is what we refer to as the "acquisition technology".

3.1 Technology

Firm i produces a unique variety using a constant returns to scale technology:

$$y_t(i) = \kappa z(i) l_t(i)^{\alpha} x_t(i)^{1-\alpha}$$

$$y_t(i) = \kappa z(i) \left(\alpha l_t(i)^{\frac{\eta-1}{\eta}} + (1-\alpha) x_t(i)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}$$

Here, $z(i)$ denotes the firm's permanent productivity, $l_t(i)$ denotes the quantity of labor it uses in time period t, and $x_t(i)$ denotes the quantity of an intermediate bundle it uses. The intermediate bundle is a CES aggregate of varieties it purchases from its suppliers.

$$x_t(i) \equiv \left(\int_{G_{i,t}} \nu_t(i,k)^{(\sigma-1)/\sigma} dk \right)^{\sigma/(\sigma-1)}$$

$G_{i,t}$ denotes the set of suppliers of firm i in time period t, and $\nu_t(i,k)$ denotes the quantity of goods purchased from supplier k. The set of suppliers is a continuum and evolves endogenously over time according to a process described in Section 3.3. Given the set of suppliers, $G_{i,t}$, the price index of firm i's intermediate bundle is given by:

$$c_t(G_{i,t}) = \left(\int_{G_{i,t}} \tilde{p}_t(k,i)^{1-\sigma} dk \right)^{1/(1-\sigma)}$$

where $\tilde{p}_t(k,i)$ denotes the price supplier k charges firm i. Setting wage as the numeraire, the marginal cost of firm i is given by:

$$mc_t(i) = \frac{c_t(i)^{1-\alpha}}{z(i)}$$

3.2 Demand

Firms sell to other firms and the representative household. We assume there are no frictions in matching with the representative household, so that all firms are exogenously connected to the representative household. Search frictions, however, do exist in matching with other firms. Thus, firms exert costly acquisition effort to acquire customers through a process described further in Section 3.3. As each customer will have a continuum of suppliers, we assume the market structure is monopolistic competition.

Let $H_{i,t}$ denote the set of firms which are customers of firm i in time period t. Again, $H_{i,t}$ is an endogenous object which evolves over time according to a process which will be described in Section 3.3. Let $p_t(i,j)$ denote the price firm i charges firm $j \in H_{i,t}$ in time period t. The production technology implies sales from firm i to firm j are given by:

$$r_t(i,j) = \left(\frac{p_t(i,j)}{c_t(G_{j,t})}\right)^{1-\sigma} m_t(j) \quad \forall j \in H_i$$

where $m_t(j) \equiv \int_{G_{j,t}} r_t(i,j)di$ denotes the total intermediate expenditure of firm j in time period t.

The representative household supplies labor inelastically and spends its entire income on a CES bundle of varieties. We assume final customers have the same elasticity of substituion across varieties as firms. Let $p_t^f(i)$ denote the price firm i charges the representative household in time period t. Final demand for firm i's variety is given by:

$$FD_t(i) = \left(\frac{p_t^f(i)}{\mathcal{P}_t}\right)^{1-\sigma} X_t$$

where $\mathcal{P}_t$ is an index of firm prices:

$$\mathcal{P}_t \equiv \left(\int p_t^f(k)^{1-\sigma}dk\right)^{\frac{1}{1-\sigma}}$$

and X_t denotes the income of the household. We assume firm profits are rebated to the household. Household income is then the sum of labor income and firm profits:

$$X_t = 1 + \int \pi_t(k)dk$$

Summing sales to downstream firms and the household, total demand for firm i is given by:

$$s_t(z(i), G_{i,t}, H_{i,t}) = \left(\frac{p_t^f(i)}{\mathcal{P}_t}\right)^{1-\sigma} X_t + \int_{H_{i,t}} \left(\frac{p_t(i,j)}{c_t(G_{j,t})}\right)^{1-\sigma} m_t(j)dj$$

3.3 Network Formation

Firms enter without any customers or suppliers. Due to the presence of search frictions, firms must exert costly acquisition effort and engage in random search to match with customers and suppliers. Specifically, in period t firm i exerts effort in finding customers $u_t(i)$ and effort in finding suppliers $v_t(i)$. Exerting effort $u_t(i)$ costs the firm $\varphi_h(u_t(i))$ units of labor and exerting effort $v_t(i)$ costs the firm $\varphi_g(v_t(i))$ units of labor. We assume $\frac{\partial \varphi_h(u)}{\partial u} > 0$ and $\frac{\partial \varphi_g(v)}{\partial v} > 0$. Importantly, we also assume $\frac{\partial^2 \varphi_h(u)}{\partial u^2} > 0$ and $\frac{\partial^2 \varphi_g(v)}{\partial v^2} > 0$. This assumption of strictly convex acquisition costs encapsulates our idea of "acquisition". It generates the feature that firms slowly acquire customers and suppliers over the lifecycle rather than jumping immediately to a steady-state.

Let $H_{i,t}(z', a')$ denote the measure of productivity z', age a' customers a firm is matched with in time period t and let $G_{i,t}(z', a')$ denote the measure of productivity z', age a' suppliers a firm is matched with in time period t.[3]

Given the acquisition efforts of all firms, the measure of new connections formed in time period t is given by an aggregate matching function $\mathcal{M}(\mathcal{U}_t, \mathcal{V}_t)$, where $\mathcal{U}_t \equiv \int u_t(k)dk$ is the aggregate measure of customer acquisition effort and $\mathcal{V}_t \equiv v_t(k)dk$ is the aggregate measure of supplier acquisition effort. Thus, exerting customer acqusition effort $u_t(i)$ results in a measure of new customers:

$$u_t(i)\frac{\mathcal{M}(\mathcal{U}_t, \mathcal{V}_t)}{\mathcal{U}_t}$$

Note here, the firm matches with a measure of new customers, rather than a discrete number. This implies that each firm is connected to a continuum of customers. Conditional on matching with a customer, the probability the customer is in some set $\mathcal{C}$ is proportional to the supplier acquisition effort of firms in that set. Thus, the measure of new customers firm i matches with in some set $\mathcal{C}$ is given by:

$$u_t(i)\frac{\mathcal{M}(\mathcal{U}_t, \mathcal{V}_t)}{\mathcal{U}_t}\frac{\int_{\mathcal{C}} v_t(k)dk}{\mathcal{V}_t}$$

[3]This is an abuse of notation, as before $G_{i,t}$ and $H_{i,t}$ were defined as sets.

Similarly, exerting effort $v_t(i)$ results in a measure of new suppliers:

$$v_t(i)\frac{\mathcal{M}\left(\mathcal{U}_t,\mathcal{V}_t\right)}{\mathcal{V}_t}$$

Again, note the firm matches with a measure of new suppliers, rather than a discrete number. This implies that each firm is connected to a continuum of suppliers. Conditional on matching with a supplier, the probability the supplier is in some set $\mathcal{C}$ is proportional to the customer acquisition effort of firms in that set. Thus, the measure of new suppliers firm i matches with in some set $\mathcal{C}$ is given by:

$$v_t(i)\frac{\mathcal{M}\left(\mathcal{U}_t,\mathcal{V}_t\right)}{\mathcal{V}_t}\frac{\int_{\mathcal{C}}u_t(k)dk}{\mathcal{U}_t}$$

At the end of every period, a share δ of existing relationships are exogenously destroyed.

The matching and separation processes imply the following laws of motion for customer and supplier networks:

$$H_{i,t}(z',a') = u_t(i)\frac{\mathcal{M}\left(\mathcal{U}_t,\mathcal{V}_t\right)}{\mathcal{U}_t}\frac{\tilde{v}_t(z',a')n_t(z',a')}{\mathcal{V}_t} + (1-\delta)H_{i,t-1}(z',a'-1) \tag{6}$$

$$G_{i,t}(z',a') = v_t(i)\frac{\mathcal{M}\left(\mathcal{U}_t,\mathcal{V}_t\right)}{\mathcal{V}_t}\frac{\tilde{u}_t(z',a')n_t(z',a')}{\mathcal{U}_t} + (1-\delta)G_{i,t-1}(z',a'-1) \tag{7}$$

where $\tilde{v}_t(z',a')$ denotes the supplier acquisition effort of (z',a') firms, $\tilde{u}_t(z',a')$ denotes the customer acquisition effort of (z',a') firms, and $n_t(z',a')$ denotes the measure of (z',a') firms in the economy. These laws of motion conjecture that all (z',a') firms make the same customer and supplier acquisition decisions. These conjectures will be true in equilibrium.

Intuitively, the measure of (z',a') customers a firm is connected to in period t is equal to the measure of new (z',a') matches made this period plus the measure of $(z',a'-1)$ customers the firm had in the previous period which survive. Similarly, the measure of (z',a') suppliers a firm is connected to in period t is equal to the measure of new (z',a') matches made this period plus the measure of $(z',a'-1)$ suppliers the firm had in the previous period which survive. At times, we will refer to the laws of motion described in 6 and 7 by:

$$H_{i,t} = \Gamma_t^h(u_t(i), H_{i,t-1})$$

$$G_{i,t} = \Gamma_t^g(v_t(i), G_{i,t-1})$$

The choice of indexing customers and suppliers by (z', a') is a deliberate one. Firms need to know several things about potential customers and suppliers when making their acquisition choices. With respect to suppliers, firms need to know the output price a given supplier will charge. This depends not only on the productivity z' of the supplier, but also on the supplier's set of suppliers. With respect to customers, firms need to know the input cost of a given customer as well as their total input expenditure. Again, this depends not only on the productivity z' of the customer, but also on the customer's customers and the customer's suppliers. However, due to our assumptions on the acquisition technology and the firm productivity process, in equilibrium, all (z', a') firms will have identical customer and supplier sets and thus identical output prices, input costs, and intermediate expenditures. They will also make identical customer and supplier acquisition efforts. Thus (z', a') serves as a sufficient type to identify customers and suppliers.

3.4 Timing

Timing in the model is as follows:

1. Measure n_t^e of new firms enter and draw permanent productivity
2. Firms pay acquisition costs and match with new customers and suppliers
3. Firms produce and sell to household and other firms
4. Share $(1 - \beta)$ of firms exit and share δ of relationships destroyed

3.5 Firm Problem

Firms maximize the discounted sum of profits. The problem of the firm is formulated in (8). A firm with talent z, previous-period customer set H_{t-1} and previous-period supplier set G_{t-1} chooses customer acquisition effort u_t, supplier acquisition effort v_t, price to charge the household p_t^f, and price to charge a (z', a) customer $p_t(z', a')$ to maximize the sum of current-period profits and value next period.

$$
\begin{aligned}
V_t(z, G_{t-1}, H_{t-1}) = \max_{u_t, v_t, p_t^f, p_t} & \frac{p_t^f - mc_t(z, G)}{p_t^f} \left(\frac{p_t^f}{\mathcal{P}_t} \right)^{1-\sigma} X_t \\
& + \int_{Z,A} \frac{p_t(z', a') - mc_t(z, G)}{p_t(z', a')} \left(\frac{p_t(z', a')}{\tilde{c}_t(z', a')} \right)^{1-\sigma} \tilde{m}_t(z', a') H_t(z', a') dz' da' \\
& - \varphi_h(u_t) - \varphi_g(v_t) + \beta V_{t+1}(z, G_t, H_t)
\end{aligned} \quad (8)
$$

$$G_t = \Gamma_t^g(v_t, G_{t-1}), \quad H_t = \Gamma_t^h(u_t, H_{t-1})$$

$$mc_t(z, G) = \frac{1}{z}\left(\int_{Z,A} \tilde{p}_t(z', a')^{1-\sigma} G_t(z', a') dz' da'\right)^{(1-\alpha)/(1-\sigma)}$$

Given the previous period customer and supplier sets, acquisition efforts u_t and v_t result in current-period sets, H_t and G_t. The construction of these sets requires conjectures about the acquisition efforts of other firms, $\tilde{u}_t(z', a')$ and $\tilde{v}_t(z', a')$. Given the set of current-period suppliers, the firm can compute its marginal cost $mc_t(z, G)$. This computation also requires conjectures about the output prices charged by other firms $\tilde{p}_t(z', a')$. Given its marginal cost and current-period customers, the firm chooses the output price to charge its customers. Notice, calculating current-period profits from charging a price $p_t(z', a')$ to a (z', a') customer requires conjectures about the customer's input cost $\tilde{c}_t(z', a')$ and intermediate expenditure $\tilde{m}_t(z', a')$. Similarly, calculating current-period profits from charging a price p_t^f to the household requires conjectures about the aggregate price index $\mathcal{P}_t$ and household income X_t.

Modeling both pricing choices and acquisition choices together introduces a potentially complicated problem. For example, in certain models, firms initially offer low prices to accumulate customers faster. Pricing choices can interact with acquisition choices in a nontrival way to shape the evolution of trading partners. Our assumptions on the network formation process provide tractability here.

First, notice that the price a firm charges does not affect the evolution of its customer or supplier network in Equations 6 and 7. This implies that the firm faces a static pricing problem and charges the standard monopolistic competition markup over its marginal cost. This feature in our model that firms use non-price actions rather than dynamic pricing to accumulate demand is in line with recent empirical evidence. Argente et al. (2023) find that markups in the consumer food sector do not systematically vary with a firm's age in a market. Fitzgerald et al. (2023) find that following successful entry into export markets, exporters have post-entry dynamics of quantities, but no post-entry dynamics of markups.

Second, the assumptions of CES and all customers having the same elasticity of substitution imply that every firm charges the same markup $\mu = \frac{\sigma}{\sigma-1}$ to all of its customers.

Taking this into account, we can rewrite the firm problem as:

$$
\begin{aligned}
V_t(z, G_{t-1}, H_{t-1}) = \max_{u_t, v_t} & \frac{\mu-1}{\mu}\left(\frac{\mu c_t(G_t)^{1-\alpha}}{z\mathcal{P}_t}\right)^{1-\sigma} X_t \\
& + \int_{Z,A} \frac{\mu-1}{\mu}\left(\frac{\mu c_t(G_t)^{1-\alpha}}{z\tilde{c}_t(z',a')}\right)^{1-\sigma} \frac{1-\alpha}{\mu}\tilde{s}_t(z',a')H_t(z',a')dz'da' \\
& - \varphi_h(u_t) - \varphi_g(v_t) + \beta V_{t+1}(z, G_t, H_t) \quad (9)
\end{aligned}
$$

$$
G_t = \Gamma_t^g(v_t, G_{t-1}), \quad H_t = \Gamma_t^h(u_t, H_{t-1})
$$

$$
c_t(G_t) = \left(\int_{Z,A}\left(\frac{\mu\tilde{c}_t(z',a')^{1-\alpha}}{z'}\right)^{1-\sigma} G_t(z',a')dz'da'\right)^{1/(1-\sigma)}
$$

A firm chooses acquisition efforts u_t and v_t to maximize the sum of discounted profits. There is no longer an explicit pricing choice as the firm charges a constant markup $\mu = \frac{\sigma}{\sigma-1}$ over its marginal costs. The firm still requires conjectures about the acquisition choices of other firms $\tilde{u}_t(z',a')$, $\tilde{v}_t(z',a')$, and the input costs of other firms $\tilde{c}_t(z',a')$, however it no longer requires an explicit conjecture about other firms' output prices. It understands all firms charge a constant markup over their marginal cost. Also, rather than requiring a conjecture about a customer's intermediate expenditure, $\tilde{m}_t(z',a')$, the firm now requires a conjecture about the customer's sales, $\tilde{s}_t(z',a')$. The firm understands that a customer's intermediate expenditure is equal to a share $\frac{1-\alpha}{\mu}$ of its sales.

Solving the firm's problem, a firm's optimal customer acquisition choice is given by:

$$
\frac{\mu-1}{\mu}\sum_{\tau=0}^{\infty}\sum_{z',a'}((1-\delta)\beta)^{\tau}\left(\frac{\mu c_{t+\tau}(G_{t+\tau})^{1-\alpha}}{z\tilde{c}_{t+\tau}(z',a'+\tau)}\right)^{1-\sigma}\frac{1-\alpha}{\mu}\tilde{s}_{t+\tau}(z',a'+\tau)\frac{\mathcal{M}(\mathcal{U}_t,\mathcal{V}_t)}{\mathcal{U}_t}\frac{\tilde{v}_t(z',a')n_t(z',a')}{\mathcal{V}_t} = \frac{\partial\varphi_h(u_t)}{\partial u_t} \quad (10)
$$

The term on the left is the marginal benefit of customer acquisition. This is equal to the discounted sum of future profits generated by (z',a') customers, multiplied by the measure of (z',a') matches an additional unit of acquisition creates. Future profits generated from a (z',a') customer depend on the path of the customer's sales, which governs their intermediate expenditure, and the path of the firm's input cost relative to the customer's input cost, which governs the firm's share in the customer's intermediate expenditure. This marginal benefit is set equal to the marginal cost of acquisition, which is the term on the right.

Equation 10 highlights how the firm's problem in our framework differs from other models of customer and supplier matching. Relative to one-sided matching models in which firms acquire customers from an exogenous set (e.g. customer capital models), a key difference here is that the set of customers is an endogenous object. For example, the endogenous supplier acquisition effort of a (z', a') firm, $\tilde{v}(z', a')$, affects the measure of matches a firm makes with (z', a') customers. In our framework, firm acquisition efforts endogenously respond to the acquisition efforts of potential partners on the other side of the market.

However, there also exist two-sided matching models in which a set of customers searches for suppliers and a set of suppliers searches for customers. In these models, firm acquisition efforts do endogenously respond to the other side of the market. Relative to to this class of models, however, a key difference here comes from firms being embedded in a network. When a firm acquires customers, this shifts out demand not only for it, but also for its suppliers. A firm's customer acquisition effort affects the profits of its suppliers and so also the customer acquisition effort of its suppliers. This can be seen through the sales of a (z', a') customer, $\tilde{s}_t(z', a')$, entering Equation 10. The fact that firms do not consider the profits of their customers when making their acquisition choices will give rise to vertical externalities which we explore further in Section 5.

$$\frac{\mu-1}{\mu}\sum_{\tau=0}^{\infty}\sum_{z',a'}((1-\delta)\beta)^{\tau}\left(\frac{\mu\tilde{c}_{t+\tau}(z',a'+\tau)^{1-\alpha}}{z'c_{t+\tau}(G_{t+\tau})}\right)^{1-\sigma}(1-\alpha)s_{t+\tau}(z,G_{t+\tau},H_{t+\tau})\frac{\mathcal{M}\left(\mathcal{U}_t,\mathcal{V}_t\right)}{\mathcal{V}_t}\frac{\tilde{u}_t(z',a')n_t(z',a')}{\mathcal{U}_t}=\frac{\partial\varphi_g(v_t)}{\partial v_t} \tag{11}$$

$$\sum_{\tau=0}^{\infty}\sum_{z',a'}((1-\delta)\beta)^{\tau}\frac{\mu-1}{\mu}s_{t+\tau}(z,G_{t+\tau},H_{t+\tau})(1-\alpha)\left(\frac{\mu\tilde{c}_{t+\tau}(z',a'+\tau)^{1-\alpha}}{z'c_{t+\tau}(G_{t+\tau})}\right)^{1-\sigma}\frac{\mathcal{M}\left(\mathcal{U}_t,\mathcal{V}_t\right)}{\mathcal{V}_t}\frac{\tilde{u}_t(z',a')n_t(z',a')}{\mathcal{U}_t}=\frac{\partial\varphi_g(v_t)}{\partial v_t}$$

$$\sum_{\tau=0}^{\infty}\sum_{z',a'}((1-\delta)\beta)^{\tau}\frac{\mu-1}{\mu}s_{t+\tau}(z,G_{t+\tau},H_{t+\tau})e_x(c_{t+\tau}(G_{t+\tau}))\left(\frac{\mu\,\tilde{m}c_{t+\tau}(z',a'+\tau)}{c_{t+\tau}(G_{t+\tau})}\right)^{1-\sigma}\frac{\mathcal{M}\left(\mathcal{U}_t,\mathcal{V}_t\right)}{\mathcal{V}_t}\frac{\tilde{u}_t(z',a')n_t(z',a')}{\mathcal{U}_t}=\frac{\partial\varphi_g(v_t)}{v_t}$$

In Equation 11, we characterize the firm's optimal supplier acquisition effort. The term on the left is the marginal benefit of supplier acquisition. This is equal to the discounted sum of future profits generated by matching with additional (z', a') suppliers, multiplied by the measure of (z', a') matches an additional unit of acquisition creates. Additional suppliers reduce the marginal cost of the firm, allowing them to charge lower prices to their customers and increase their profits. This marginal benefit is equated to the marginal cost of acquisition, which is the term on the right.

Similar comments as above can be made on how the firm problem here differs from other models of customer and supplier matching. Again, the key mechanisms are that acquisition efforts of firms respond endogenously to potential partners on the other side of the market; and that acquisition efforts affect the profits of upstream and downstream partners, though firms do not internalize this when choosing these efforts.

We emphasize here that the presence of constant markups does not preclude competition among suppliers. As seen through Equation 11, when firms choose their supplier acquisition effort, they consider how the output price they offer compares to the input cost of their customers. In other words, how their output price compares to the output prices offered by other suppliers. This informs their decision to reduce their marginal cost through acquiring suppliers. Firms in this setting compete with eachother through prices, but the channel through which they do this is through the marginal costs they attain rather than the markups they charge.

3.6 Equilibrium

We now define an equilibrium.

Definition 1 *An equilibrium is defined as equilibrium objects:*

1. *optimal policies* $\{u_t(z, G, H),\ v_t(z, G, H)\}_{t=0}^{\infty}$
2. *conjectures about other agents:*
   (a) *acquisition efforts:* $\{\tilde{u}_t(z, a), \tilde{v}_t(z, a)\}_{t=0}^{\infty}$
   (b) *input cost and sales:* $\{\tilde{c}_t(z, a)\}, \tilde{s}_t(z', a')\}_{t=0}^{\infty}$
   (c) *customer and supplier sets:* $\{\tilde{H}_{z,a,t}(z', a'), \tilde{G}_{z,a,t}(z', a')\}_{t=0}^{\infty}$
3. *conjectures about aggregate objects:*
   (a) *aggregate price index* $\{\mathcal{P}_t\}_{t=0}^{\infty}$
   (b) *aggregate income* $\{X_t\}_{t=0}^{\infty}$

which satisfy the following conditions:

1. *Optimal policies solve firm problem 9.*
2. *Conjectures about customer and supplier sets of other firms are consistent with laws of motion:*

$$\tilde{H}_{z,a,t} = \Gamma_t^h\left(\tilde{u}_t(z, a), \tilde{H}_{z,a-1,t-1}\right), \quad \tilde{G}_{z,a,t}(z', a') = \Gamma_t^g\left(\tilde{v}_t(z, a), \tilde{G}_{z,a-1,t-1}\right)$$

3. *Conjectures about acquisition efforts of other firms are consistent with optimal policies:*

$$\tilde{u}_t(z, a) = u_t(z, \tilde{G}_{z,a-1,t-1}, \tilde{H}_{z,a-1,t-1}), \quad \tilde{v}_t(z, a) = v_t(z, \tilde{G}_{z,a-1,t-1}, \tilde{H}_{z,a-1,t-1})$$

4. Conjectures about input costs and sales are consistent with customer and supplier sets:

$$\tilde{c}_t(z,a) = c_t(\tilde{G}_{z,a,t}), \quad \tilde{s}_t(z,a) = s_t\left(z, \tilde{G}_{z,a,t}, \tilde{H}_{z,a,t}\right)$$

5. Conjectures about aggregate objects are consistent with optimal policies:

$$\mathcal{P}_t = \left(\int \left(\mu z(i)^{-1} c_t(G_{it})^{1-\alpha}\right)^{1-\sigma} di\right)^{1/(1-\sigma)}, \quad X_t = 1 + \frac{\mu - 1}{\mu} \int s_t\left(z(i), G_{it}, H_{it}\right) di$$

We now define a stationary equilibrium. For the remainder of this paper, we will focus on stationary equilibria.

Definition 2 *A stationary equilibrium is defined as an equilibrium in which equilibrium objects are time-independent.*

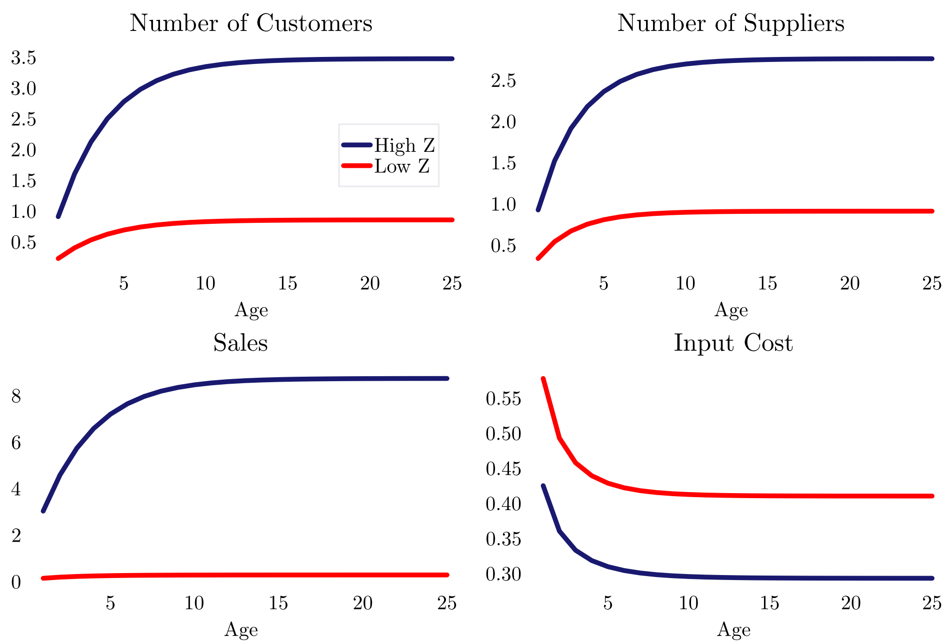
Even in a static setting, solving for equilibria in endogenous network models is a complicated problem. A firm's choice to form links depends on payoff-relevant attributes of potential trading partners. However, these attributes of trading partners depend themselves on the attributes of their own trading partners. Thus in equilibrium, one needs to solve for two fixed points. First, a fixed point in payoff-relevant attributes, as the attributes of a firm depend on the attributes of its partners. Second, a fixed point in the link formation choices of firms, as the link formation choices depend on the attributes of potential trading partners. The endogenous production network literature has made progress here by considering environments in which these fixed points can be easily characterized and solved for.

Our setting further complicates this problem by introducing dynamics in customer and supplier networks. As in the static setting, a firm's decision to form links depends on the payoff-relevant attributes of potential trading partners. However, as trading partners themselves acquire customers and suppliers over the lifecycle, their payoff-relevant attributes evolve over time. Furthermore as the payoff-relevant attributes of trading partners depend on the attributes of their own trading partners, the evolution of their attributes depends on how their trading partners' attributes evolve over time. Again in equilibrium, we need to find a fixed-point in attributes and a fixed point in link formation choices, but here as firms evolve over time, it cannot be a fixed point in attributes and choices of individual firms. Instead, given the acquisition technology, we need to find a state space such that all firms in the same state have the same attributes and make the same choices. Then in equilibrium, we find fixed points in the attributes and choices of different states. The assumptions we make provide a state, (z, a), such that these fixed points can be easily characterized and solved for.[4]

3.7 Model Output: Lifecycle Patterns

In Figure 7 we plot lifecycle trajectories from the calibrated version of the model. We discuss the calibration procedure in the next section, but for now we point out the fact that the model is able to generate the patterns we observe in the data. Older firms have more customers and suppliers, lower output prices and input costs, and greater sales and intermediate expenditure.

Figure 7: Model Output



Note: We plot lifecycle trajectories of firms in the calibrated model. Our calibration strategy is described in Section 4. In the calibrated model, there are two productivity types.

The fact that the model is able to generate these patterns should not be very surprising. The assumption of a convex acquisition cost implies that firms slowly acquire customers and suppliers over the lifecycle, rather than jumping immediately to a steady-state set. The patterns look similar to output one would get from any "customer capital" model. However, a key difference here is that firms in our setting are embedded in an endogenous network. Thus, the acquisition choices they make will have important implications for their upstream and downstream partners, and the structure of the production network. The usefulness of our framework is in studying these implications.

[4] In Appendix B.1, I describe the exact solution method.

4 Structural Estimation

In this section, we estimate the structural parameters of our model using simulated method of moments. The goal is to use the model to quantitatively study how customer and supplier acquisition shapes aggregate productivity.

We first impose functional forms. We assume a standard form for the acquisition cost:

$$\varphi_g(v) = \frac{\xi}{\zeta} v^{\zeta}, \quad \varphi_h(u) = \frac{\xi}{\zeta} u^{\zeta}$$

The level parameter ξ governs the level of costs, while the curvature parameter ζ governs how easily firms can scale their customer and supplier networks. We impose that the level and curvature parameters are the same for both customer and supplier acquisition.

We assume that the permanent productivity of a firm can take two values. Upon entry, a firm draws a productivity of 1 with probability p_{low}, and draws a permanent productivity of $\bar{z} > 1$ with probability $(1 - p_{low})$

$$F(z) = \begin{cases} 1 \text{ with prob. } p_{low} \\ \bar{z} \text{ with prob. } (1 - p_{low}) \end{cases}$$

Finally, we assume that the aggregate matching function is Cobb-Douglas, where γ governs the elasticity of matches with respect to customer acquisition.

$$\mathcal{M}(\mathcal{U}, \mathcal{V}) = \mathcal{U}^{\gamma} \mathcal{V}^{1-\gamma}$$

This results in nine parameters which require estimation. The model parameters are listed in Table 1. Most of these parameters are common to many models and so are calibrated in standard ways. The two which are less common are the level of acquisition cost and curvature of acquisition cost. We start by discussing how these parameters affect equilibrium objects in our setting. This will inform our strategy for estimating them.

4.1 How do acquisition costs affect equilibrium objects?

We first explore the role of the level parameter ξ. Proposition 1 states that changing the level parameter from ξ_0 to ξ_1 uniformly scales equilibrium acquisition efforts of firms by a factor $\left(\frac{\xi_1}{\xi_0}\right)^{-1/\zeta}$. As a result, the equilibrium sales, input costs, number of customers, and number of suppliers all scale by constant factors. However, as all firms scale by the same factor, there are no changes in patterns across the lifecycle nor across the cross-section of

Table 1: Model Parameters

Parameter	Description
α	labor share
σ	elasticity of substitution
γ	matching function elasticity
β	exogenous firm survival rate
δ	exogenous separation rate
z_{high}	productivity of high type
p_{low}	probability of low type
ξ	level of acquisition cost
ζ	curvature of acquisition cost

firms. In our calibration, we will normalize the level parameter to $\xi = 1$, understanding that we can use Proposition 1 to calculate how the level of aggregate objects would differ for alternative values of ξ.

Proposition 1 *Suppose acquisition policies in an equilibrium with acquisition cost $\varphi_0(x) = \frac{\xi_0}{\zeta} x^\zeta$ are given by:*

$$u(z, H_{-1}, G_{-1}), \quad v(z, H_{-1}, G_{-1})$$

In an equilibrium with cost function $\varphi_1(x) = \frac{\xi_1}{\zeta} x^\zeta$ (holding all other parameters fixed), equilibrium acquisition policies are given given by:

$$\left(\frac{\xi_1}{\xi_0}\right)^{-1/\zeta} u(z, H_{-1}, G_{-1}), \quad \left(\frac{\xi_1}{\xi_0}\right)^{-1/\zeta} v(z, H_{-1}, G_{-1})$$

Next, we explore the role of the curvature paramameter ζ. Taking the ratio of first order conditions (i.e. Equation 10) for two firms i and i', we can express their relative customer acquisition efforts in Equation 12:

$$\frac{u_{i,t}}{u_{i',t}} = \left[\frac{\frac{\mu-1}{\mu} \sum_{s=0}^{\infty} \sum_{z',a'} ((1-\delta)\beta)^s \left(\frac{\mu c(G_{i,t+s})^{1-\alpha}}{z_i \tilde{c}(z',a'+s)} \right)^{1-\sigma} \frac{1-\alpha}{\mu} \tilde{s}(z', a'+s) \frac{\mathcal{M}(\mathcal{U},\mathcal{V})}{\mathcal{U}} \frac{\tilde{v}(z',a') n(z',a')}{\mathcal{V}}}{\frac{\mu-1}{\mu} \sum_{s=0}^{\infty} \sum_{z',a'} ((1-\delta)\beta)^s \left(\frac{\mu c(G_{i',t+s})^{1-\alpha}}{z_{i'} \tilde{c}(z',a'+s)} \right)^{1-\sigma} \frac{1-\alpha}{\mu} \tilde{s}(z', a'+s) \frac{\mathcal{M}(\mathcal{U},\mathcal{V})}{\mathcal{U}} \frac{\tilde{v}(z',a') n(z',a')}{\mathcal{V}}} \right]^{\frac{1}{\zeta-1}} \tag{12}$$

The relative acquisition efforts of firms i and i' depends on the ratio of expected profits generated from matching with additional customers. Notice the only difference in customer acquisition effort across firms arise from differences in marginal cost. Firms who expect to have lower marginal costs today and in the future exert more effort in acquiring customers, as conditional on matching with a customer, they sell more. The curvature of the cost function, ζ, governs the elasticity between marginal costs and acquisition efforts. When ζ is low, small differences in marginal costs lead to large differences in acquisition effort. On the other hand, when ζ is high, all firms similar acquisition choices despite large differences in marginal costs. In other words, ζ governs the extent to which acquisition effort responds to technical advantages.

As ζ governs the elasticity between marginal costs and customer acquisition effort, it also governs the correlation between number of customers and sales. When ζ is low, firms with different marginal costs, and thus different average sales, will make very different acquisition choices, and thus have very different numbers of customers. This generates a high correlation between the number of customers a firm has and its sales. On the other hand, when ζ is high, firms with different marginal costs, and thus different average sales, will make very similar acquisition choices, and thus have very similar numbers of customers. This generates a low correlation between the number of customers a firm has and its sales. Thus, we use the correlation between number of customer and sales to calibrate the curvature parameter. Furthermore, an advantage of using this moment is that it has been reported for other countries. In particular, Arkolakis et al. (2023) use the same moment to calibrate an acquisition cost curvature using data from Chile. In Section 6, we will use this moment to study how differences in acquisition technology shape differences in aggregate productivity.

4.2 Estimation Procedure

We assign existing estimates to a subset of our parameters. We use the labor share from the 2019 Penn World Table to set $\alpha = 0.52$. We set $\sigma = 4.30$, referring to Baqaee et al. (2023). We set $\gamma = 0.5$, which is in line with Krolikowski and McCallum (2021).

The remaining parameters are jointly estimated to match moments in our firm-to-firm data. Though all parameters are jointly estimated, there exists an intuitive mapping between the parameters and target moments. We set exogenous survival rate $\beta = 0.66$ to match the 1-year exit rate of sellers. We set exogenous separation rate $\delta = 0.32$ to match the 1-year survival rate of connections. As discussed in Section 4.1, we normalize ξ such that the average number of customers is equal to 1, and estimate ζ by targeting the elasticity between number of customers and sales. In particular, we compute this elasticity by 4-digit HSN and use the average across HSNs. This moment in our data is 0.36. Finally, we estimate the productivity of the high type $\bar{z}$ and probability of drawing the low type p_{low} by targeting the interquartile range and skewness of log sales. Again, we compute these by 4-digit HSN and use the average across HSNs.

We summarize our estimation results in Table 2. We match all moments well.

Table 2: Estimation Results

Parameter	Value	Target	Target Moment	Model Moment
α	0.52	Penn World Table 2019	-	-
σ	4.30	Baqaee et al. (2023)	-	-
γ	0.50	Krolikowski and McCallum (2021)	-	-
β	0.66	1-year exit rate	0.33	0.33
δ	0.32	1-year survival rate of connections	0.45	0.45
$\bar{z}$	2.08	IQR of log sales	2.99	2.98
p_{low}	0.59	skewness of log sales	0.39	0.39
ξ	0.59	normalization (average NC = 1)	-	-
ζ	3.10	elasticity between NC and Sales	0.36	0.36

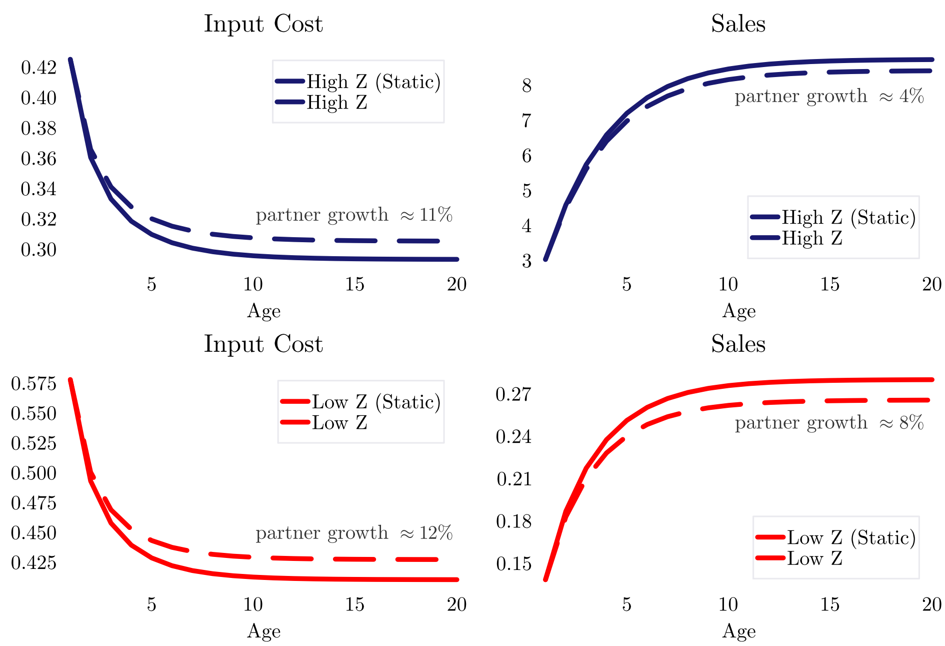
4.3 Firm Dynamics in a Network Model

Before studying our main research questions, we briefly discuss how acquisition choices of firms interact through the network to shape firm dynamics. This differentiates our setting from the "customer capital" literature.

As noted in Section 3.7, the calibrated model generates the lifecycle patterns we observe in the data. Older firms have more customers and suppliers, greater sales and intermediate expenditure, and lower output prices and input costs. However as firms are embedded in a network here, the growth in their sales and the decline in their input costs no longer depend just on their own acquisition choices. As a firm's customer itself acquires more customers, it shifts out its sales and so also its demand for intermediates. This leads to the customer buying more intermediate from the firm. At the same time, as a firm's customer acquires more suppliers, the firm becomes less competitive relative to the customer's supplier network. This leads to the customer spending a lower share of intermediate expenditure on the firm. Finally, as a firm's supplier acquires more suppliers, it reduces its marginal cost and passes this cost reduction onto the firm. Thus, the dynamics of firm sales and input costs depend not only on the firm's acquisition decisions, but also on the acquisition decisions of its customers and suppliers.

In Figure 8, we decompose lifecycle dynamics into an "own growth" channel and a "partner growth" channel. Specifically, we plot counterfactual trajectories for a case in which firms make the same acquisition choices and draw partners from the same distribution, but after matching with a partner, the partner no longer evolves over time. The counterfactual trajectories are plotted in the dashed lines in the figure, while the solid lines correspond to the equilibrium trajectories in Figure 7. In the static partner counterfactual, the decline in input costs is 11% lower for high productivity firms and 12% lower for low productivity firms. Furthermore, sales growth for high productivity firms is 4% lower, while it is 8% lower for low productivity firms. Figure 8 highlights an important mechanism which shapes firm dynamics in network models. A firm's growth in a network depends on its own choices, but also on the choices of its trading partners.

Figure 8: Dynamics Contribution of Trading Partners



Note: We decompose lifecycle trajectories into an "own growth" channel and a "partner growth" channel. The solid lines correspond to the equilibrium trajectories. The dashed lines correspond to counterfactuals in which firms make the same acquisition efforts and draw from the partners from the same distribution, but partners no longer evolve over time after matching.

5 Efficiency

In this section, we study the normative properties of our environment. We find that the decentralized equilibrium is inefficient as firms fail to internalize vertical externalities and search externalities. First, we describe the problem of a planner who chooses production choices for firms, taking the production network as given. Next, we describe the full problem of a planner who chooses both production and acquisition choices for firms, imposing that the planner must use the same acquisition technology as firms in the decentralized economy. Proceeding in this manner helps distinguish inefficiencies which arise in exogenous network models from those that arise in endogenous network models. We find inefficient production and acquisition choices generate quantitatively large aggregate losses. Aggregate productivity in the efficient allocation is 16% greater than in the decentralized equilibrium. Furthermore, changes in allocative efficiency will be central for understanding how technology differences map to productivity differences, as will be discussed in Section 6.

5.1 Efficiency of Exogenous Network Equilibrium

We first study the problem of a planner who chooses production choices for firms, taking the production network as given. The exogenous network can be described as a collection of customer and supplier sets $\{H_{z,a}, G_{z,a}\}_{z,a}$. The problem of the planner is to choose consumption of varieties, $d(z,a)$, labor input of firms, $l(z,a)$, and intermediate inputs of firms, $\nu(z,a,z',a')$, to maximize aggregate consumption. Here, $\nu(z,a,z',a')$, denotes the quantity of a (z',a') variety a (z,a) firm uses. The planner's choices are subject to firm output and aggregate labor constraints.

$$\max_{d,l,\nu} \left(\sum_{z,a} d(z,a)^{\frac{\sigma-1}{\sigma}} n(z,a) \right)^{\frac{\sigma}{\sigma-1}}$$

$$\text{s.t.} \qquad d(z,a) + \sum_{z',a'} \nu(z',a',z,a) H_{z,a}(z',a') \leq y(z,a) \quad \forall z,a$$

$$\sum l(z,a) n(z,a) \leq L$$

$$y(z,a) = \kappa z l(z,a)^{\alpha} \left(\sum_{z',a'} \nu(z,a,z',a')^{(\sigma-1)/\sigma} G_{z,a}(z',a') \right)^{\frac{\sigma(1-\alpha)}{\sigma-1}}$$

$$\{H_{z,a}, G_{z,a}\}_{z,a} \quad \text{Given}$$

The first constraint imposes that the sum of final consumption of a variety and usage of it by other firms in production must be less than the output of that variety. The second constraint imposes that aggregate labor input must be less than aggregate labor supply.

To test whether the decentralized equilibrium is efficient, we can compare the decentralized choices to the choices which satisfy the planner's first order condition. In the case the decentralized equilibrium is efficient, these two choices will coincide. We find that the decentralized choices for consumption and labor input match the planner's choices, however intermediate input usage does not. In Equation 13, we express the decentralized firm's input demand. In Equation 14, we express the choice which satisfies the planner's first order condition.

$$\nu(z,a,z',a') = \left(\frac{\mu c(z',a')^{1-\alpha}}{z'}\right)^{-\sigma} c(z,a)^{\sigma-1}\frac{1-\alpha}{\mu}s(z,a) \tag{13}$$

$$\nu(z,a,z',a') = \mu^{\sigma}\left(\frac{\mu c(z',a')^{1-\alpha}}{z'}\right)^{-\sigma} c(z,a)^{\sigma-1}\frac{1-\alpha}{\mu}s(z,a) \tag{14}$$

Comparing the planner's choice to the decentralized choice, we see that the two diverge. Firms in the decentralized equilibrium underutilize intermediate inputs relative to the planner's allocation. The reason for this inefficiency is that firms fail to internalize a "vertical externality" and set prices inefficiently high (multiple marginalization). The price a supplier charges its customer affects the customer's marginal cost and thus the customer's profit. However the supplier does not take this into account when setting its price. In addition, the price the customer charges its own customers affects its demand, and thus the demand and profits of the supplier. However, again, the customer does not take this into account when setting its price. As a result, in equilibrium, the price for intermediate inputs is set inefficiently high and firms underutilize intermediate inputs.

It is useful to compare the setting here to a similar setting which lacks intermediate input usage. Suppose firms still sell to the household, but no longer sell to other firms nor use intermediate inputs. If labor supply is inelastic, then this modified setting is efficient. Constant markups preclude distortions between firms. The inelasticity of labor supply precludes the option of underproduction. In contrast, in the exogenous network setting, firms underproduce due to underutilization of intermediate inputs. The point is that market power is not enough in this setting to generate inefficiency. This inefficiency requires both market power and intermediation chains.

5.2 Efficiency of Endogenous Network Equilibrium

We next study the efficiency of an endogenous network equilibrium. As in the exogenous network problem, the planner chooses consumption of varieties $d(z,a)$, labor input $l(z,a)$, and intermediate inputs $\nu(z,a,z',a')$, to maximize aggregate consumption. In addition, the planner also chooses customer acquisition effort $u(z,a)$, and supplier acquisition effort $v(z,a)$. The production network then arises out of the same acquisition technology as in the decentralized equilibrium. Again, we can describe the network as a collection of customer and supplier sets $\{H_{z,a}, G_{z,a}\}_{z,a}$. However, here, these sets are endogenous objects which arise out of the planner's acquisition efforts.

$$\max_{d,l,\nu,u,v} \left(\sum_{z,a} d(z,a)^{\frac{\sigma-1}{\sigma}} n(z,a) \right)^{\frac{\sigma}{\sigma-1}}$$

$$d(z,a) + \sum_{z',a'} \nu_{z',a'}(z,a) H_{z,a}(z',a') \leq y(z,a) \quad \forall z,a$$

$$\sum_{z,a} l(z,a)n(z,a) + \sum_{z,a} \varphi_h(u(z,a))n(z,a) + \sum_{z,a} \varphi_g(v(z,a))n(z,a) \leq L$$

$$y(z,a) = \kappa z l(z,a)^{\alpha} \left(\sum_{z',a'} \nu(z,a,z',a')^{(\sigma-1)/\sigma} G_{z,a}(z',a') \right)^{\frac{\sigma(1-\alpha)}{\sigma-1}}$$

$$H_{z,a}(z',a') = u(z,a) \frac{\mathcal{M}(\mathcal{U},\mathcal{V})}{\mathcal{U}} \frac{v(z',a')n(z',a')}{\mathcal{V}} + (1-\delta) H_{z,a-1}(z',a'-1) \quad \forall z,a,z',a'$$

$$G_{z,a}(z',a') = v(z,a) \frac{\mathcal{M}(\mathcal{U},\mathcal{V})}{\mathcal{V}} \frac{u(z',a')n(z',a')}{\mathcal{U}} + (1-\delta) G_{z,a-1}(z',a'-1) \quad \forall z,a,z',a'$$

The first constraint imposes that the sum of final consumption of a variety and usage of it by other firms in production must be less than the output of that variety. The second constraint imposes that aggregate labor input must be less than aggregate labor supply. Different from the exogenous network problem, however, here aggregate labor input is the sum of labor used for production and labor used for acquisition. The third constraint is just the production function of the firm. The last two constraints impose that the planner uses the same acquisition technology as firms in the decentralized equilibrium.

The planner's first order conditions with respect to consumption, labor input, and intermediate input usage are identical to the exogenous network problem, so we omit further discussion, and instead focus on the first order conditions with respect to customer and supplier acquisition. Let $\lambda(z,a)$ denote the shadow value of a (z,a) variety and W denote the shadow wage. The planner's choice for customer acquisition effort $u(\hat{z},\hat{a})$ is given by:

$$\sum_{z,a}\sum_{s=0}^{\infty}(1-\delta)^s\left(\lambda(z,a+s)\frac{\partial y(z,a+s)}{\partial G_{z,a+s}(\hat{z},\hat{a}+s)}v(z,a)n(\hat{z},\hat{a})-\lambda(\hat{z},\hat{a}+s)\nu(z,a+s,\hat{z},\hat{a}+s)v(z,a)n(z,a)\right)\frac{\mathcal{M}\left(\mathcal{U},\mathcal{V}\right)}{\mathcal{U}\mathcal{V}}$$
$$+\frac{\partial\frac{\mathcal{M}(\mathcal{U},\mathcal{V})}{\mathcal{U}}}{\partial u(\hat{z},\hat{a})}\frac{1}{\frac{\mathcal{M}(\mathcal{U},\mathcal{V})}{\mathcal{U}}}\left[\Gamma_1-\Gamma_2\right]=W\frac{\partial\varphi_h(u(\hat{z},\hat{a}))}{\partial u(\hat{z},\hat{a})}n(\hat{z},\hat{a})$$

$$\Gamma_1=\sum_{z,a}\sum_{z',a'}\lambda(z,a)\frac{\partial y(z,a)}{\partial G_{z,a}(z',a')}G_{z,a}(z',a')$$

$$\Gamma_2=\sum_{z,a}\sum_{z',a'}\lambda(z,a)\nu(z',a',z,a)H_{z,a}(z',a')$$

The first term on the left is the marginal surplus generated by a $(\hat{z},\hat{a})$ firm connecting with more customers. This term is equal to the increase in customers' output due to adding a $(\hat{z},\hat{a})$ supplier, multiplied by the value of that output, minus the value of $(\hat{z},\hat{a})$ output used in production by the customers. The second term on the left is the marginal effect firm $(\hat{z},\hat{a})$ has on all other matches through congestion in the matching function. The planner equates the sum of these two terms to the marginal cost of acquisition, which is the the term on the right.

The planner's choice for supplier acquisition effort $v(\hat{z},\hat{a})$ is given by:

$$\sum_{z,a}\sum_{s=0}^{\infty}(1-\delta)^s\left(\lambda(\hat{z},\hat{a}+s)\frac{\partial y(\hat{z},\hat{a}+s)}{\partial G_{\hat{z},\hat{a}+s}(z,a+s)}u(z,a)n(z,a)-\lambda(z,a+s)\nu(\hat{z},\hat{a}+s,z,a+s)u(z,a)n(\hat{z},\hat{a})\right)\frac{\mathcal{M}\left(\mathcal{U},\mathcal{V}\right)}{\mathcal{U}\mathcal{V}}$$
$$+\frac{\partial\frac{\mathcal{M}(\mathcal{U},\mathcal{V})}{\mathcal{V}}}{\partial v(\hat{z},\hat{a})}\frac{1}{\frac{\mathcal{M}(\mathcal{U},\mathcal{V})}{\mathcal{V}}}\left[\Gamma_1-\Gamma_2\right]=W\frac{\partial\varphi_g(v(\hat{z},\hat{a}))}{\partial v(\hat{z},\hat{a})}n(\hat{z},\hat{a})$$

The first term on the left is the marginal surplus generated by a firm $(\hat{z},\hat{a})$ connecting with more suppliers. This term is equal to the increase in firm $(\hat{z},\hat{a})$'s output due to adding suppliers, multiplied by the value of that output, minus the value of the additional intermediate inputs firm $(\hat{z},\hat{a})$ now uses. The second term on the left is the marginal effect firm $(\hat{z},\hat{a})$ has on all matches through congestion in the matching function. The planner equates the sum of these two terms to the marginal cost of acquisition, which is the the term on the right.

As before, to test whether decentralized acquisition effort is efficient, we can compare the decentralized choice to the choice which satisfies the planner's first order condition. In the case the decentralized choice is efficient, the two will coincide. In Equation 15, we express the decentralized customer acquisition effort for a (z,a) firm. In Equation 16, we express the acquisition effort which satisfies the planner's first order condition.

$$\frac{\mu-1}{\mu}\sum_{\tau=0}^{\infty}\sum_{z',a'}((1-\delta)\beta)^{\tau}\left(\frac{\mu c(z,a+\tau)^{1-\alpha}}{zc(z',a'+\tau)}\right)^{1-\sigma}\frac{1-\alpha}{\mu}s(z',a'+\tau)\frac{\mathcal{M}(\mathcal{U},\mathcal{V})}{\mathcal{U}}\frac{v(z',a')n(z',a')}{\mathcal{V}}=\frac{\partial\varphi_h(u)}{\partial u} \quad (15)$$

$$\frac{\mu^2-1}{\mu}\sum_{z',a'}\sum_{\tau=0}^{\infty}((1-\delta)\beta)^{\tau}\left(\frac{\mu c(z,a+\tau)^{1-\alpha}}{zc(z',a'+\tau)}\right)^{1-\sigma}\frac{1-\alpha}{\mu}s(z',a'+\tau)\frac{\mathcal{M}(\mathcal{U},\mathcal{V})}{\mathcal{U}}\frac{v(z',a')n(z',a')}{\mathcal{V}}$$
$$+\frac{\partial\frac{\mathcal{M}(\mathcal{U},\mathcal{V})}{\mathcal{U}}}{\partial u}\frac{1}{\frac{\mathcal{M}(\mathcal{U},\mathcal{V})}{\mathcal{U}}}\frac{\mathcal{P}}{\mu}[\Gamma_1-\Gamma_2]=\frac{\partial\varphi_h(u)}{\partial u} \quad (16)$$

In Equation 17, we express the decentralized supplier acquisition effort for a (z,a) firm. In Equation 18, we express the acquisition effort which satisfies the planner's first order condition.

$$\frac{\mu-1}{\mu}\sum_{\tau=0}^{\infty}\sum_{z',a'}((1-\delta)\beta)^{\tau}\left(\frac{\mu c(z',a'+\tau)^{1-\alpha}}{z'c(z,a+\tau)}\right)^{1-\sigma}(1-\alpha)s(z,a+\tau)\frac{\mathcal{M}(\mathcal{U},\mathcal{V})}{\mathcal{V}}\frac{u(z',a')n(z',a')}{\mathcal{U}}=\frac{\partial\varphi_g(v)}{\partial v} \quad (17)$$

$$\frac{\mu^2-1}{\mu^2}\sum_{z',a'}\sum_{\tau=0}^{\infty}((1-\delta)\beta)^{\tau}\left(\frac{\mu c(z',a'+\tau)^{1-\alpha}}{z'c(z,a+\tau)}\right)^{1-\sigma}(1-\alpha)s(z,a+\tau)\frac{\mathcal{M}(\mathcal{U},\mathcal{V})}{\mathcal{V}}\frac{u(z',a')n(z',a')}{\mathcal{U}}$$
$$+\frac{\partial\frac{\mathcal{M}(\mathcal{U},\mathcal{V})}{\mathcal{V}}}{\partial\mathcal{V}}\frac{1}{\frac{\mathcal{M}(\mathcal{U},\mathcal{V})}{\mathcal{V}}}\frac{\mathcal{P}}{\mu}[\Gamma_1-\Gamma_2]=\frac{\partial\varphi_g(v)}{\partial v} \quad (18)$$

The planner's optimal choices differs from the decentralized firm's for two reasons. First, the planner internalizes congestion externalities which arise through the matching function. This is the second term on the left in Equations 16 and 18. The planner understands that a (z,a) firm's customer acquisition effort affects matching probabilities for other firms searching for customers, and takes this into account when choosing acquisition effort for the firm. Similarly, the planner understands that a (z,a) firm's supplier acquisition effort affects matching probabilities for other firms searching for suppliers, and takes this into account when choosing acquisition effort for the firm.

Secondly, firms in the decentralized equilibrium fail to internalize a vertical externality[5]. When a decentralized firm exerts more acquisition effort, this creates more matches. These matches generate surplus for the firm, but also generate surplus for the partners it

[5]Though we label this externality a "vertical" externality, there are other labels which have been used for it. In the labor literature, this externality has been labeled "thickness" and "composition" externalities. In some of the endogenous network literature, this externality has been labeled the "match creation" externality.

matches with. However, decentralized firms only internalize their private surplus, while the planner internalizes joint surplus. The vertical externality can be seen in the first term on the left when comparing Equations 15 and 16, and when comparing Equations 17 and 18.

In the case the congestion and vertical externalities exactly offset eachother for all firms in the decentralized equilibrium, the decentralized acquisition efforts are efficient. However under the decentralized surplus-splitting protocol, this does not happen and so acquisition efforts are inefficient.

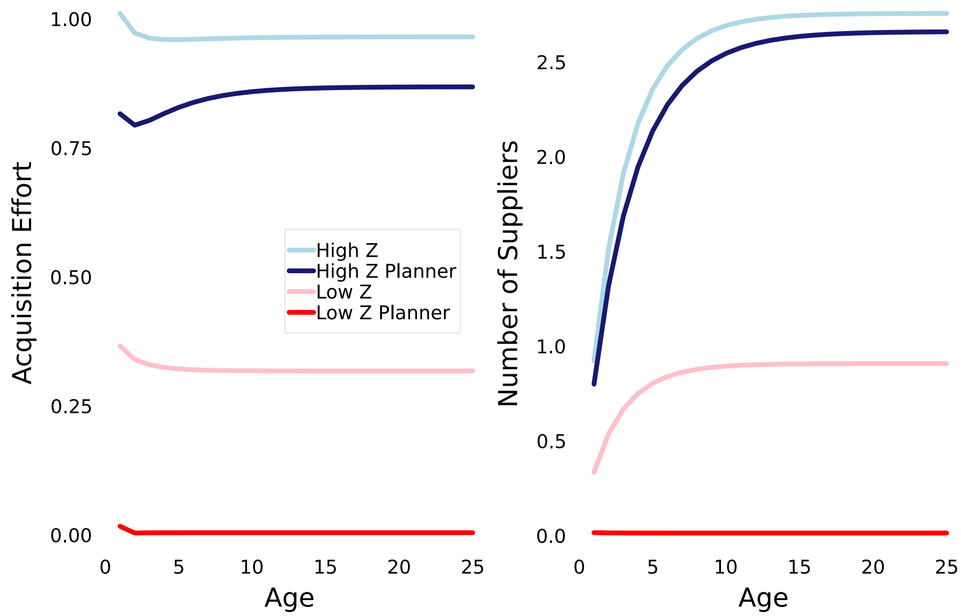
5.3 Efficient Allocation

We discuss the efficient allocation in this section. We find large productivity losses from inefficient production and acquisition choices. Aggregate productivity is 16% greater in the efficient allocation, relative to the decentralized equilibrium.

In Figure 9, we compare efficient supplier acquisition to decentralized supplier acquisition. The left panel displays acquisition efforts. All firms exert too much effort in acquiring suppliers. This is especially true for low-productivity firms. While acquisition effort in the efficent allocation is roughly 20% lower for high-productivity firms, low-productivity firms in the efficient allocation almost entirely stop acquiring suppliers. The results of these acquisition efforts are shown in the right panel, which displays the trajectories for number of suppliers over the lifecycle. The trajectory for high-productivity firms remains largely unchanged, while low-productivity firms in the efficient allocation have virtually no suppliers. Notice, despite exerting less effort in acquiring customers in the efficient allocation, high-productivity firms maintain the same number of suppliers. This is because there is less congestion in matching due to reduced acquisition efforts of firms, especially of low-productivity firms.

In Figure 10, we compare efficient customer acquisition to decentralized customer acquisition. The left panel displays acquisition efforts. The magnitude of acquisition effort for high-productivity firms is roughly similar, however young firms in the decentralized equilibrium exert relatively more acquisition effort than the efficient level. This difference reflects the fact that the planner's acquisition efforts are more correlated with the marginal costs of firms, which decline over time. Again, low-productivity firms almost entirely stop acquiring customers. The results of these efforts are displayed in the right panel, which displays trajectories for number of customers over the lifecycle. The trajectory of number of customers for high-productivity firms in the efficient allocation is similar to the decentralized trajectory. Low-productivity firms, on the other hand, have virtually no customers in the efficient allocation.

Figure 9: Efficient Supplier Acquisition

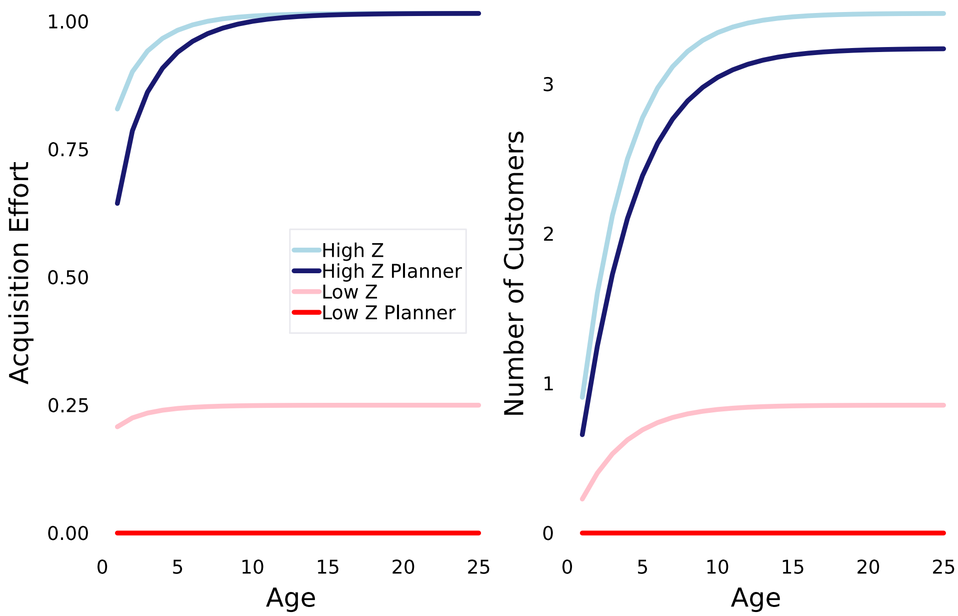


Note: The left panel displays the decentralized equilibrium supplier acquisition efforts and the efficient acquisition efforts. The right panel plots the lifecycle trajectories of number of suppliers for firms in the decentralized equilibrium and the efficient allocation.

The efficient acquisition efforts give rise to the efficient production network. In Figure 11, we compare the efficient network to the decentralized network. We plot the difference in the share of connections between given supplier-customer pairs. The horizontal axis displays the type of the supplier, with the left half corresponding low-productivity suppliers of various ages, and the right half corresponding to high-productivity suppliers. The vertical axis displays the type of the customer, with the bottom half corresponding to low-productivity customers, and the top half corresponding to high-productivity customers. For each gridpoint in the figure, a positive value indicates that the efficient network has a greater share of connections between these types, while a negative value indicates that the efficient network has a lesser share. The efficient network has a greater share of connections between high-productivity suppliers and high-productivity customers.

Finally, in Figure 12, we compare the efficient input choices to the decentralized choices. In the left panel, we display labor input choices. Low-productivity firms in the efficient allocation use significantly less labor input as they shrink in size. On the other hand, high-productivity firms use significantly more labor in the efficient allocation. Part of this increase comes from labor being reallocated away from low-productivity firms, however this does not explain the entire increase. As firms exert less acquisition effort overall in the efficient

Figure 10: Efficient Customer Acquisition



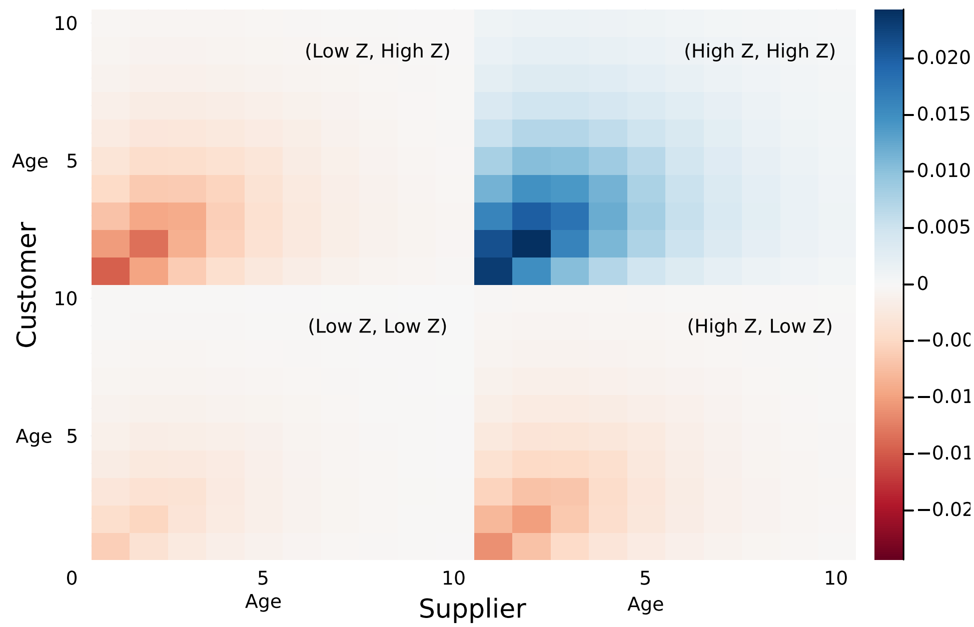
Note: The left panel displays the decentralized equilibrium customer acquisition efforts and the efficient acquisition efforts. The right panel plots the lifecycle trajectories of number of customers for firms in the decentralized equilibrium and the efficient allocation.

allocation, there is more labor available for production. In particular, whereas 86% of labor is used in production in the decentralized equilibrium, 92% of labor is used in production in the efficient allocation. This increase in the labor available for production is used by high-productivity firms.

In the right panel, we display the quantity of intermediate input used. In the efficient allocation, high-productivity firms use significantly more intermediate input. First, the efficient allocation lacks a distortion from multiple marginalization. In addition, high-productivity firms, despite having similar numbers of customers and suppliers, are matched on average with higher productivity partners. On the other hand, intermediate input usage falls for low-productivity firms. Despite the lack of a distortion from multiple marginalization and being matched with higher productivity partners on average, the steep reduction in their number of partners leads to a fall in their intermediate input usage.

We find large aggregate productivity differences between the efficient allocation and the decentralized equilibrium. Final output is 16% greater in the efficient allocation. As the stock of labor is inelastic here, this gain entirely reflects an increase in productivity stemming from the better coordination of acquisition and production choices.

Figure 11: Efficient Network vs. Decentralized Network



Note: We plot the difference between the decentralized production network and the efficient production network. The horizontal axis corresponds to low-productivity (left half) and high-productivity (right half) suppliers of varying ages. The vertical axis corresponds to low-productivity (bottom half) and high-productivity (top half) customers of varying ages. Each gridpoint displays the difference between the share of connections between the corresponding types in the efficient allocation and the decentralized equilibrium. A positive value indicates that the efficient network has a greater share of connections between the corresponding types, while a negative value indicates that the efficient network has a lesser share.

6 Acquisition Technology and Aggregate Productivity

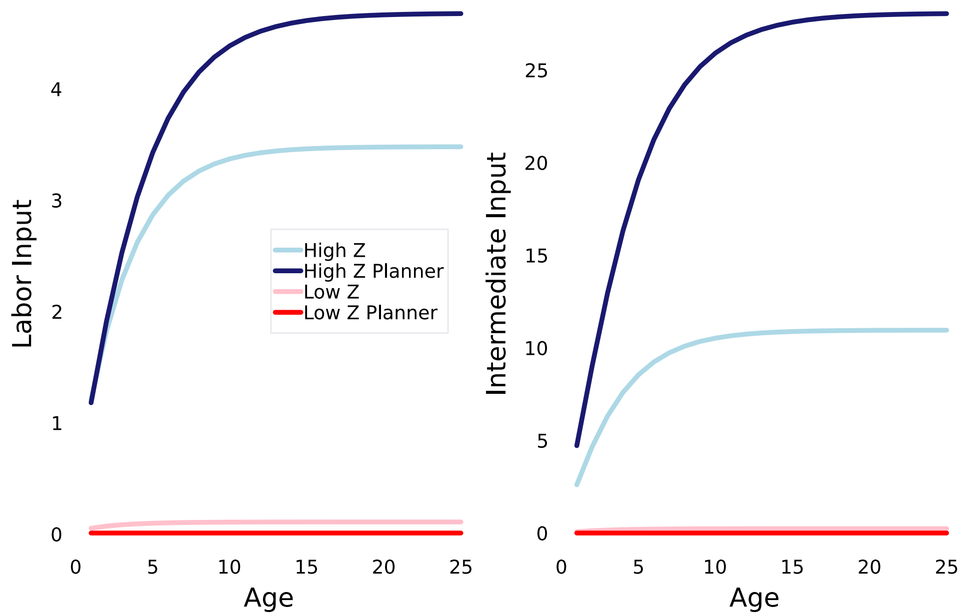
In this section, we study how differences in acquisition technology map to aggregate productivity differences. In our model, the acquisition technology is governed by the parameters of the acquisition cost. Thus, we study comparative statics with these parameters.

We begin by studying the role of the curvature of acquisition costs ζ. As discussed in Section 4.1, ζ is disciplined by the elasticity between number of customers and sales. This moment, however, is also reported in Arkolakis et al. (2023) for Chile. Arkolakis et al. (2023) estimate this elasticity to be 0.42 in Chile[6], compared to the elasticity of 0.36 we find in India. Through the lens of our model, these moments imply that Chilean firms are able to scale their customer and supplier networks more easily than Indian firms.

In a counterfactual exercise, we recalibrate ζ to match the Chilean moment. So that the total number of connections remain constant, we also adjust the level of acquisition costs. All other parameters are held fixed. The counterfactual calibration is summarized in Table

[6] Again, similar to our moment, this is an average across sectors.

Figure 12: Efficient Inputs



Note: The left panel plots the lifecycle trajectories of labor input for firms in the decentralized equilibrium and the efficient allocation. The right panel plots the lifecycle trajectories of intermediate input for firms in the decentralized equilibrium and the efficient allocation.

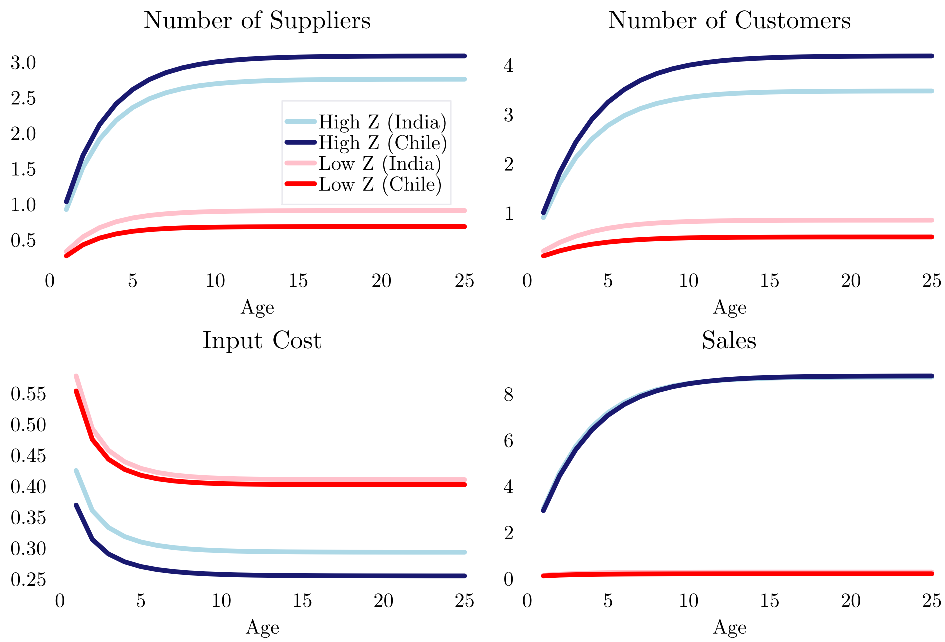
3. The elasticity between number of customers and sales implies $\zeta = 2.50$ in Chile, compared to $\zeta = 3.10$ in India. This estimate of 2.50 is is similar to the (average) curvature estimated in Arkolakis et al. (2023).

Table 3: Chile ζ Counterfactual

Parameter	India Parameter	Chile Parameter	Target	India Moment	Chile Moment
α	0.52	0.52	Penn World Table 2019	-	-
σ	4.30	4.30	Baqaee et al. (2023)	-	-
γ	0.50	0.50	Krolikowski and McCallum (2021)	-	-
β	0.66	0.66	1-year exit rate	0.33	0.33
δ	0.32	0.32	1-year survival rate of connections	0.45	0.45
z_{high}	2.08	2.08	IQR of log sales	2.98	3.21
p_{low}	0.59	0.59	skewness of log sales	0.39	0.40
ξ	0.59	0.41	normalization (average NC = 1)	-	-
ζ	3.10	2.50	Elasticity between NC and Sales	0.36	0.42

In Figure 13, we compare the lifecycle trajectories of firms in the Chilean counterfactual to lifecycle trajectories of firms in India. As can be seen in the top right panel of Figure 13, high-productivity firms enter with a greater number of customers and grow to larger steady state in the counterfactual economy, while low-productivity firms enter with fewer customers and grow to a smaller steady state. Similarly, the top left panel reveals that high-productivity firms in the counterfactual economy enter with more suppliers and grow to larger steady state, while low-productivity firms enter with fewer suppliers and grow to a smaller steady state. As discussed in Section 4.1, ζ governs the elasticity between marginal cost and customer acquisition effort. As ζ is lower in Chile, differences in marginal costs generate larger differences in customer acquisition effort. A similar mechanism has analogous effects on supplier acquisition effort. As a result, high productivity firms expand relative to low productivity firms in the counterfactual economy.

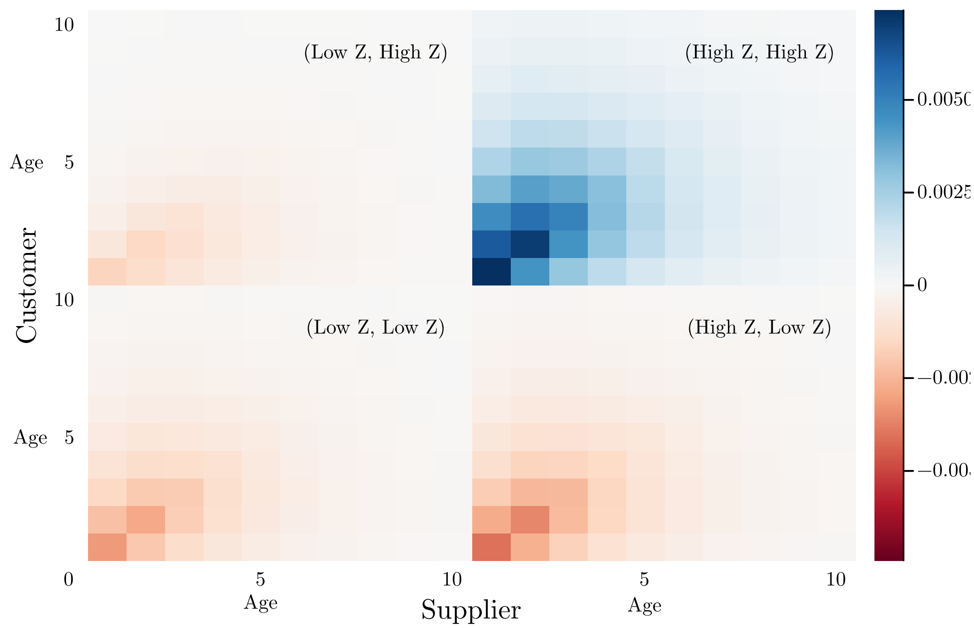
Figure 13: Lifecycle Trajectories of Chilean Firms



Note: We plot lifecycle trajectories for firms in our baseline economy, and for firms in the counterfactual economy calibrated to match the Chilean moment.

As firms here are embedded in a network, their acquisition choices will have important implications for the production network. In Figure 14, we document how the structure of the production network in the counterfactual economy differs from the baseline. The figure plots the difference in the share of connections between given supplier-customer pairs. The horizontal axis displays the type of the supplier, with the left half corresponding low-productivity suppliers of various ages, and the right half corresponding to high-productivity suppliers. The vertical axis displays the type of the customer, with the bottom half corresponding to low-productivity customers, and the top half corresponding to high-productivity customers. For each gridpoint in the figure, a positive value indicates that the counterfactual network has a greater share of connections between these types, while a negative value indicates that the counterfactual network has a lesser share. The counterfactual network has a greater share of connections between high-productivity suppliers and high-productivity customers, and a lesser share between all other pairs. The greater dispersion between high and low-productivity firms in acquisition choices generates a network in which connections are more concentrated in high-productivity supplier-customer pairs.

Figure 14: Production Network of Chile vs. India



Note: We plot the difference between the production network in the baseline economy and the network in the counterfactual economy calibrated to match the Chilean moment. The horizontal axis corresponds to low-productivity (left half) and high-productivity (right half) suppliers of varying ages. The vertical axis corresponds to low-productivity (bottom half) and high-productivity (top half) customers of varying ages. Each gridpoint displays the difference between the share of connections between the corresponding types in the counterfactual economy and the baseline economy. A positive value indicates that the counterfactual economy has a greater share connections between the corresponding types, while a negative value indicates that the counterfactual economy has a lesser share.

The acquisition choices of firms and the production network determine the sales and input costs of firms. In the bottom right panel of Figure 13, we show that input costs fall for both high-productivity and low-productivity firms in the Chilean counterfactual. The decline in input costs for high-productivity firms makes sense. High-productivity firms in the counterfactual economy have more suppliers and are matched with better suppliers on average. However, the decline in input costs for low-productivity firms is more surprising. Low-productivity firms realize cost reductions, despite being matched with fewer suppliers, due to their suppliers being of higher productivity on average. Notice in the counterfactual economy, not only are firms directly connected to higher productivity suppliers, but now their suppliers too are connected to higher productivity suppliers. Thus, at every length of supply chain, firms are connected on average to higher productivity suppliers. Differences in acquisition choices of firms are amplified through the network to shape input costs.

The bottom left panel displays equilibrium (nominal) sales for firms. Both low and high-productivity firms see their sales fall. Low-productivity firms have fewer customers and are competing with better suppliers, as high-productivity suppliers now make up a greater share of any given customer's supplier network. This leads to their sales declining. Sales also fall for high-productivity firms. Despite the fact high-productivity firms have more customers, they are also competing with better suppliers. This composition effect dominates a pure number of customers effect.

We find that aggregate productivity in India would be 3.1% greater if Indian firms could scale trading partners as easily as firms in Chile. As the total number of connections is held fixed, this difference is due to the production network being more concentrated in links between high-productivity firms.

As discussed in Section 5, the decentralized equilibrium is inefficient. Thus changes in technology affect aggregate productivity through affecting both technical efficiency, i.e. productivity in the efficient allocation, and allocative efficiency, i.e. distance of the decentralized allocation from the efficient allocation. In Table 4, we compare aggregate productivity under the Indian technology and the Chilean technology, for both the decentralized and efficient allocations. We normalize by aggregate productivity in the Indian decentralized allocation. The second column displays productivities in the decentralized allocations, while the third column displays productivities in the efficient allocations. The last column computes the percentage difference between the efficient and decentralized allocations for each technology. The last row computes the percentage difference between the two decentralized allocations and the two efficient allocations.

We find that the productivity differences between the efficient allocations are much smaller than those between the decentralized allocations. While aggregate productivity in the Chilean decentralized allocation is 3.1% greater than the Indian decentralized allocation, aggregate productivity in the Chilean efficient allocation is only 0.6% greater than the Indian efficient allocation. This implies that most of the productivity gain realized in our counterfactual comes from an improvement in allocative efficiency, rather than technical efficiency. Roughly 4/5 of the change in aggregate productivity is due to allocative efficiency, while 1/5 is due to technical efficiency. In other words, though the efficient frontier does not shift significantly under the Chilean technology, firms in the decentralized economy are much closer to the efficient frontier. Aggregate productivity in the efficient allocation is 13% greater than in the decentralized allocation under the Chilean technology, compared to 16% under the Indian technology.

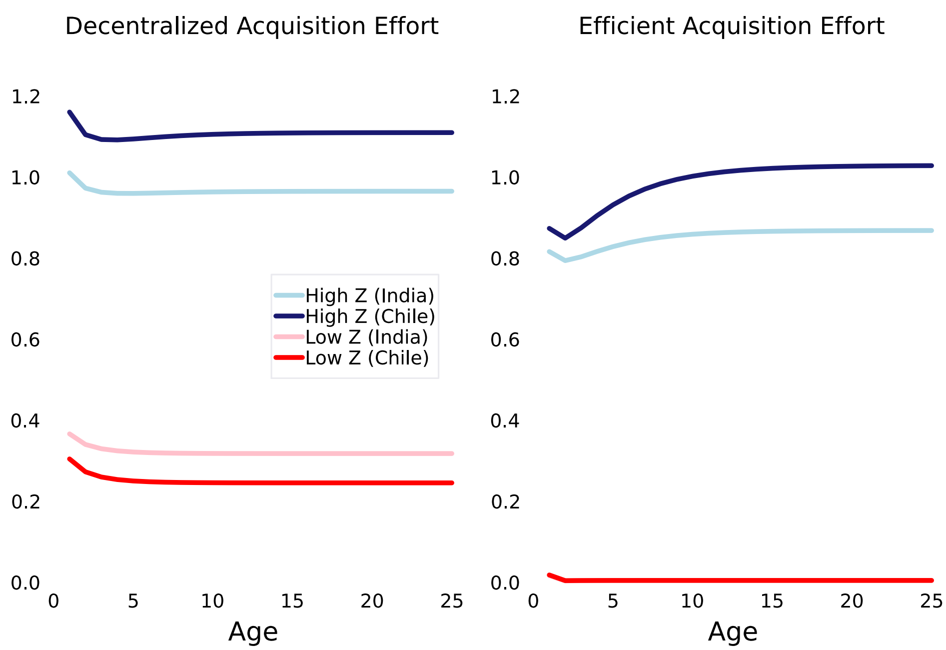
Table 4: Technical vs. Allocative Efficiency

Technology	Decentralized Productivity	Efficient Productivity	% Difference
India	100.0	115.6	15.6
Chile	103.1	116.3	12.9
% Difference	3.1	0.6	

Note: This table compares aggregate productivity under the Indian and Chilean technologies. The first column displays productivity in the decentralized allocations under both technologies. The second column displays productivity in the efficient allocations under both technologies. We normalize by productivity in the Indian decentralized allocation. The last column displays the percentage difference between the efficient and decentralized economies for each technology. The last row displays the difference between the two decentralized allocations and the two efficient allocations.

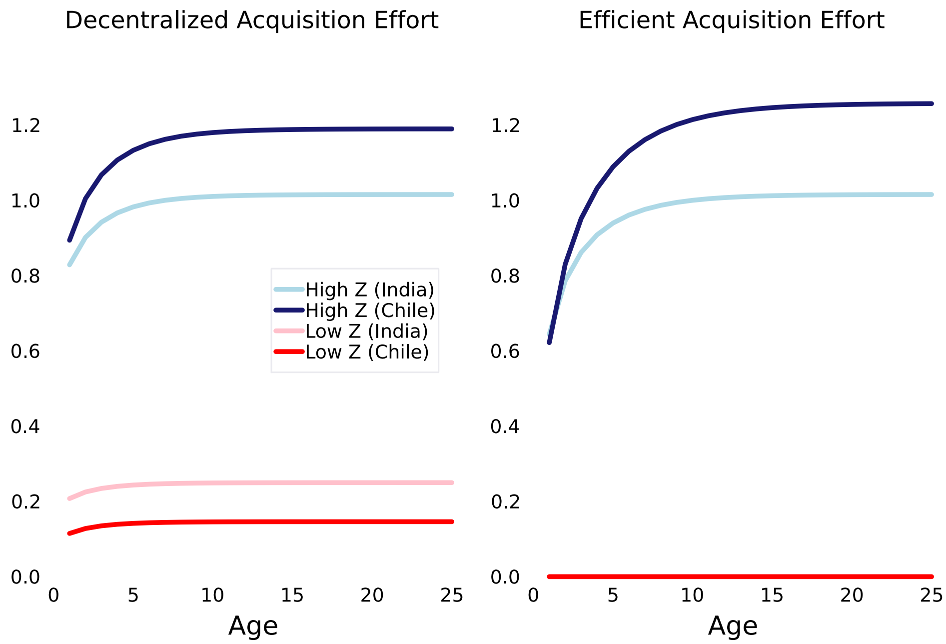
Why is the decentralized allocation closer to the efficient frontier under the Chilean technology? In Figure 15 and Figure 16, we compare the decentralized and efficient acquisition efforts under both technologies. Under both technologies, the planner achieves aggregate productivity gains by shutting out low-productivity firms. This leads to production networks where only high-productivity firms are present. However, under the Chilean technology, as the curvature of acquisition costs is lower than in India, low-productivity firms make less acquisition effort and are less present in the decentralized production network. Thus, there are smaller gains from shutting them out.

Figure 15: Efficient Supplier Acquisition: Chile vs. India



We now turn to the role of the level of acquisition costs ξ. As a corollary of Proposition 1, we derive an expression which relates differences in ζ to differences in aggregate output.

Figure 16: Efficient Customer Acquisition: Chile vs. India



Corollary 1 *Suppose final output in an equilibrium with acquisition cost* $\varphi_0(x) = \frac{\xi_0}{\zeta}x^\zeta$ *is:*

$$Y_0$$

Final output in an equilibrium with acquisition cost $\varphi_1(x) = \frac{\xi_1}{\zeta}x^\zeta$ *(holding all other parameters fixed) is given by:*

$$\frac{Y_1}{Y_0} = \left(\frac{\xi_1}{\xi_0}\right)^{-\frac{1-\alpha}{\zeta\alpha(\sigma-1)}}$$

Corollary 1 states that changing the level of acquisition costs (holding all other parameters fixed) results in an output difference which depends on the size of the level change and model parameters. Furthermore, the elasticity of aggregate output with respect to ξ is decreasing in the curvature of acquisition costs, ζ, labor share, α, and elasticity of substition, σ. When ζ is greater, acquisition efforts are less elastic and so respond less to changes in the level of acquisition costs. When α is greater, intermediates are less important in production. As a result, gains from variety, generated by changes in ξ, matter less for output. Finally, when σ is higher, there are less gains from variety and so changes in the number of suppliers generate smaller changes in output. Using the calibrated parameters for ζ, α, and σ, we find that a 10% reduction in the level of acquisition costs in India leads to a 1.0% increase in aggregate output.

7 Conclusion

In this paper, we study how customer and supplier acquisition affects aggregate productivity through shaping the production network. Using firm-to-firm transaction data from a large Indian state, we document that younger firms have fewer customers and suppliers, lower sales and intermediate expenditures, and higher input costs and output prices. Motivated by these patterns, we develop a tractable model of endogenous network formation where firms undertake costly acquisition of customers and suppliers over the lifecycle. We show that the decentralized equilibrium is inefficient due to the presence of vertical and search externalities, and that inefficient pricing and acquisition choices lead to quantitatively large aggregate productivity losses. We use the model to study how differences in acquisition technology map to aggregate productivity differences. When firms can scale trading partners more easily, high-productivity firms expand acquisition effort relative to low-productivity firms. As a result, the production network features a greater share of connections between high-productivity firms, generating higher aggregate productivity. Improvements in allocative efficiency play a central role in generating these productivity gains.

References

Abowd, J. M., F. Kramarz, and D. N. Margolis (1999). High wage workers and high wage firms. *Econometrica 67*(2), 251–333.

Acemoglu, D. and P. D. Azar (2020). Endogenous production networks. *Econometrica 88*(1), 33–82.

Afrouzi, H., A. Drenik, and R. Kim (2023). Concentration, market power, and misallocation: The role of endogenous customer acquisition.

Aker, J. C. (2010). Information from markets near and far: Mobile phones and agricultural markets in niger. *American Economic Journal: Applied Economics 2*(3), 46–59.

Argente, D., D. Fitzgerald, S. Moreira, and A. Priolo (2023). How do firms build market share?

Arkolakis, C. (2010). Market penetration costs and the new consumers margin in international trade. *Journal of Political Economy 118*(6), 1151–1199.

Arkolakis, C., F. Huneeus, and Y. Miyauchi (2023). Spatial production networks.

Atalay, E., A. Hortacsu, J. Roberts, and C. Syverson (2011). Network structure of production. *Proceedings of the National Academy of Sciences 108*(13), 5199–5202.

Baqaee, D., A. Burstein, C. Duprez, and E. Farhi (2023). Supplier churn and growth: A micro-to-macro analysis.

Baqaee, D. R. and E. Farhi (2020). Productivity and misallocation in general equilibrium. *The Quarterly Journal of Economics 135*(1), 105–163.

Bernard, A. B., E. Dhyne, G. Magerman, K. Manova, and A. Moxnes (2022). The origins of firm heterogeneity: A production network approach. *Journal of Political Economy 130*(7), 1765–1804.

Bernard, A. B., A. Moxnes, and Y. U. Saito (2019). Production networks, geography, and firm performance. *Journal of Political Economy 127*(2), 639–688.

Boehm, J. (2022). The impact of contract enforcement costs on value chains and aggregate productivity. *Review of Economics and Statistics 104*(1), 34–50.

Boehm, J. and E. Oberfield (2020). Misallocation in the market for inputs: Enforcement and the organization of production. *The Quarterly Journal of Economics 135*(4), 2007–2058.

Boehm, J., R. South, E. Oberfield, and M. Waseem (2024). The network origins of firm dynamics: Contracting frictions and dynamism with long-term relationships.

Brancaccio, G., M. Kalouptsidi, T. Papageorgiou, and N. Rosaia (2023). Search frictions and efficiency in decentralized transport markets. *The Quarterly Journal of Economics 138*(4), 2451–2503.

Cavenaile, L., M. A. Celik, J. Perla, and P. Roldan-Blanco (2023). A model of product awareness and industry life cycles. Technical report.

Chaney, T. (2014). The network structure of international trade. *American Economic Review 104*(11), 3600–3634.

Demir, B., A. C. Fieler, D. Y. Xu, and K. K. Yang (2023). O-ring production networks. *Journal of Political Economy, forthcoming*.

Dhyne, E., A. K. Kikkawa, M. Mogstad, and F. Tintelnot (2021). Trade and domestic production networks. *The Review of Economic Studies 88*(2), 643–668.

Dinlersoz, E. M. and M. Yorukoglu (2012). Information and industry dynamics. *American Economic Review 102*(2), 884–913.

Drozd, L. A. and J. B. Nosal (2012). Understanding international prices: Customers as capital. *American Economic Review 102*(1), 364–395.

Eaton, J., D. Jinkins, J. R. Tybout, and D. Xu (2022). Two-sided search in international markets.

Einav, L., P. J. Klenow, J. D. Levin, and R. Murciano-Goroff (2022). Customers and retail growth.

Fitzgerald, D., S. Haller, and Y. Yedid-Levi (2023). How exporters grow. *Review of Economic Studies, forthcoming*.

Foster, L., J. Haltiwanger, and C. Syverson (2016). The slow growth of new plants: Learning about demand? *Economica 83*(329), 91–129.

Gourio, F. and L. Rudanko (2014). Customer capital. *Review of Economic Studies 81*(3), 1102–1136.

Goyal, A. (2010). Information, direct access to farmers, and rural market performance in central india. *American Economic Journal: Applied Economics 2*(3), 22–45.

Hottman, C. J., S. J. Redding, and D. E. Weinstein (2016). Quantifying the sources of firm heterogeneity. *The Quarterly Journal of Economics 131*(3), 1291–1364.

Huneeus, F. (2020). Production network dynamics and the propagation of shocks.

Jensen, R. (2007). The digital provide: Information (technology), market performance, and welfare in the south indian fisheries sector. *The quarterly journal of economics 122*(3), 879–924.

Kopytov, A., B. Mishra, K. Nimark, and M. Taschereau-Dumouchel (2023). Endogenous production networks under supply chain uncertainty.

Krolikowski, P. M. and A. H. McCallum (2021). Goods-market frictions and international trade. *Journal of International Economics 129*, 103411.

Lim, K. (2018). Endogenous production networks and the business cycle.

Luttmer, E. G. J. (2006). *Consumer search and firm growth*.

Miyauchi, Y. (2018). Matching and agglomeration: Theory and evidence from japanese firm-to-firm trade.

Oberfield, E. (2018). A theory of input–output architecture. *Econometrica 86*(2), 559–589.

A Empirical Appendix

A.1 Bias of Coefficients under Endogenous Mobility

In estimating firm input costs and output prices, we estimate the following equation:

$$e_{ij,hsn} = \psi_{i,hsn} + \phi_{j,hsn} + \varepsilon_{ij,hsn}$$

For the estimates of $\psi_{i,hsn}$ and $\phi_{j,hsn}$ to be unbiased, we require:

$$\mathbb{E}\left[s_i' \varepsilon_{ij}\right] = 0 \quad \forall i$$

$$\mathbb{E}\left[d_j' \varepsilon_{ij}\right] = 0 \quad \forall j$$

where $S = [s_1, s_2, ..., s_N]$ is the seller fixed effects design matrix and $D = [d_1, d_2, ..., d_N]$ is the customer fixed effects design matrix. These conditions are also known in the labor literature as the assumption of exogenous mobility (Abowd et al. (1999)). In this context, the conditions imply that for every supplier, the average match-specific effect, ε_{ij}, is zero across customers; and that for every customer, the average match-specific effect is zero across suppliers. This requires firms to match in a manner uncorrelated with the match-specific effects.

Suppose that instead firms match on match-specific effects. For example, suppose customers take into account both the supplier fixed effect and match-specific effect when matching with a supplier. In this case, customers match with suppliers that have high supplier effects or those that have high match-specific effects for them. This means that a supplier with a lower supplier effect requires a higher match-specific effect to match with a customer. This selection implies that OLS estimates of low supplier effects will be biased upwards.

Similarly, suppose suppliers take into account both the customer fixed effect and match-specific effect when matching with a supplier. In this case, suppliers match with customers which have high customer effects or those who have high match-specific effects for them. This means that a customer with a lower customer effect requires a higher match-specific effect to match with a supplier. This selection implies that OLS estimates of low customer effects will be biased upwards.

For example suppose customers take into account both the supplier fixed effect and match-specific effect when choosing to match with a supplier.

Thus under "enogenous mobility" selection on match specific effects induce an upward bias on OLS estimates of low fixed effects. OLS estimates attenuate the true disparity in customer and supplier effects. This implies in the case of "endogenous mobility", our estimates underestimate that the true differences in input costs and output prices between young and old firms.

A.2 Variation in Unit Values

In this section, we decompose variation in unit values. The waybills which comprise our transaction data also have entries for units and quantities. Examples of units include "boxes", "kilograms", and "bales". This information is not required by the tax authority, so it is frequently missing in the waybills.

Let $r_{i,j,hsn,unit,y,m}$ denote the value of sales between supplier i and customer j within HSN hsn and unit $unit$ in year y and month m. Let $q_{i,j,hsn,unit,y,m}$ denote the corresponding quantity of good shipped. Here i and j refer, respectively, to the Tax IDs of the customer and supplier. Different from our main analysis, here HSN hsn refers to an 8-digit category. This comes at the cost of losing a 60% of our sample, as the Tax Authority only requires reporting up to the 4-digit level. However, the benefit is that we are able to compute unit values at a narrower level. Define the unit value supplier i charges customer j in an HSN-unit-year-month as:

$$p_{i,j,hsn,unit,y,m} \equiv \frac{r_{i,j,hsn,unit,y,m}}{q_{i,j,hsn,unit,y,m}}$$

Let $\bar{p}_{hsn,unit,y,m}$ denote the average unit value charged by all suppliers in a given HSN-unit-year-month.

$$\bar{p}_{hsn,unit,y,m} \equiv \frac{1}{N_{hsn,unit,y,m}} \sum_{i,j} p_{i,j,hsn,unit,y,m}$$

We estimate the following regression equation:

$$log\left(p_{i,j,hsn,unit,y,m} - \bar{p}_{hsn,unit,y,m}\right) = \psi_{i,hsn,unit,y,m} + \varepsilon_{i,j,hsn,unit,y,m} \tag{19}$$

That is, we project demeaned unit values onto supplier-hsn-unit-year-month fixed effects. In Table 5 we report R^2 values from the estimated model. The estimated model has an R^2 of 0.72, implying that most of the variation in unit values can be explained by the supplier fixed effects. In other words, variation within supplier across customers plays a small role in explaining the total variation in unit values.

Table 5: Model Fit

	N	R^2	Adjusted R^2
$log\left(p_{i,j,hsn,unit,y,m}\right)$	2,321,128	0.72	0.69

A.3 Lifecycle Patterns controlling for Firm Size

In Section 2, we document that older firms have more customers and suppliers, greater sales and intermediate expenditure, and lower input costs and output prices. These lifecycle patterns are in line with a theory of firm dynamics in which firms slowly grow to their steady-state size due to frictions in firm-to-firm matching. However, these patterns could also arise in a model in which the customer and supplier networks of a firm are determined period-by-period by the firm's idiosyncratic productivity, combined with a positive correlation between age and idiosyncratic productivity (e.g. due to survivorship bias or "learning-by-doing"). In this case, the lifecycle patterns just reflect a positive correlation between age and idiosyncratic productivity, and matching frictions do not play an important role in shaping firm dynamics. In this section, we repeat the empirical analysis of Section 2, controlling for firm sales. We find that, controlling for sales, younger firms have fewer customers and lower output prices. Furthermore, controlling for sales, younger firms have fewer suppliers, higher input costs, and lesser intermediate expenditure. These patterns suggest that frictions in firm-to-firm matching play an important role in shaping firm dynamics.

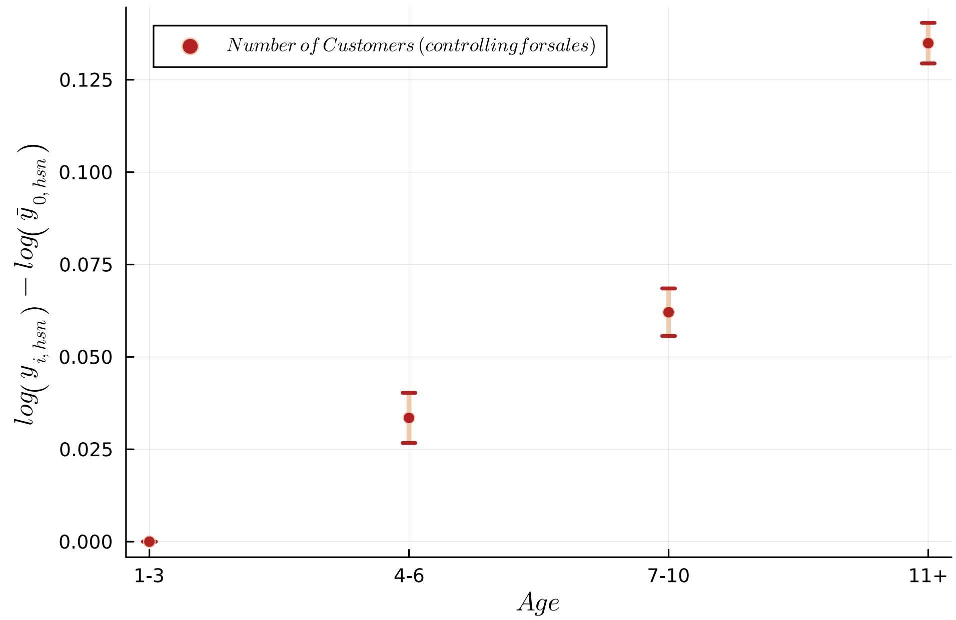
A.3.1 Downstream Facts

In Figure 17, we plot lifecycle patterns of number of customers, controlling for sales. In particular, we estimate Equation 20 and plot γ_a, normalizing by the youngest age group. The normalized estimates express the difference in log number of customers for firms in a given age category relative to entrants who sell in the same HSN and have the same sales. In the case that matching frictions do not affect firm dynamics so that number of customers and sales are just a function of firm productivity, age should not matter, after controlling for sales. However Figure 17 shows that, conditional on having the same sales, younger firms have fewer customers.

$$log\left(NC_{i,hsn}\right) = \sum_a \gamma_a \mathbf{1}(age_i \in a) + \sum_h \beta_h log\left(Sales_{i,hsn}\right)\mathbf{1}(hsn = h) + \sum_h \gamma_h \mathbf{1}(hsn = h) + \varepsilon_{i,hsn} \tag{20}$$

In Figure 18, we plot lifecycle patterns of $\psi_{i,hsn}$ from Equation 3, controlling for sales. In particular, we estimate Equation 21 and plot γ_a, normalizing by the youngest age group. The normalized estimates express the difference in $\psi_{i,hsn}$ for a given age category relative

Figure 17: Number of Customers controlling for Sales



Note: We plot estimated age fixed effects, γ_a, from Equation 20 along with bootstrap standard errors, normalizing by the youngest age group. The normalized estimates express the difference in log number of customers (within an HSN) for firms in a given age category relative to entrants who sell products in the same HSN and have the same sales.

to entrants who sell products in the same HSN and have the same sales. Noting that $\psi_{i,hsn} = (1-\sigma)log\left(\frac{p_{i,hsn}}{q_{i,hsn}}\right)$, Figure 18 shows that, conditional on having the same sales, younger firms charge lower output prices.

$$\psi_{i,hsn} = \sum_a \gamma_a \mathbf{1}(age_i \in a) + \sum_h \beta_h log\left(Sales_{i,hsn}\right)\mathbf{1}(hsn = h) + \sum_h \gamma_h \mathbf{1}(hsn = h) + \varepsilon_{i,hsn} \tag{21}$$

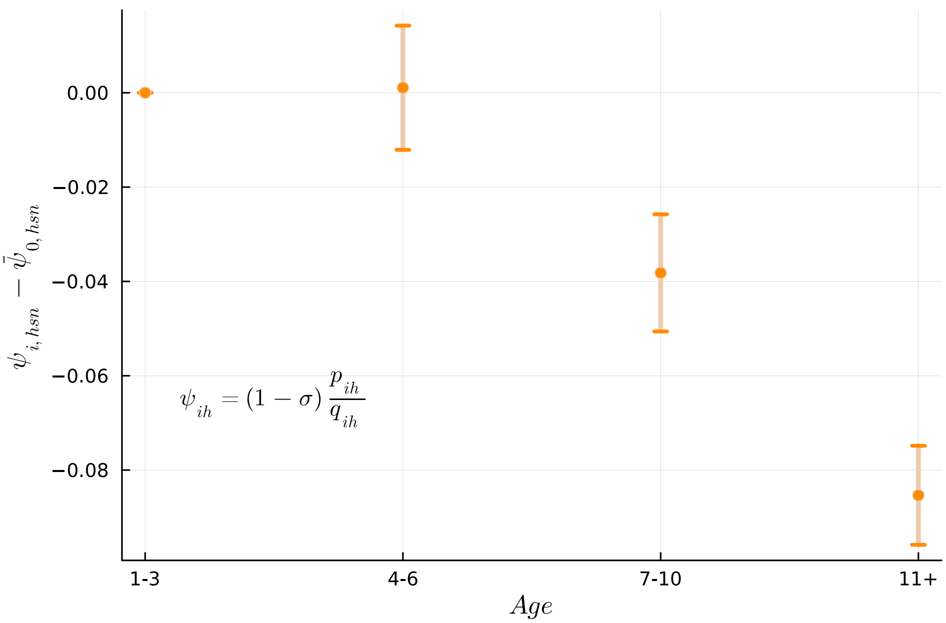
Taken together, the lifecycle patterns documented in Figure 17 and Figure 18 imply that younger firms maintain the same level of sales, despite having fewer customers, due to charging lower output prices and commanding higher average sales.

A.3.2 Upstream Facts

For each variable of interest y_{i,hsn^u}, we estimate the following regression equation:

$$log\left(y_{i,hsn^u}\right) = \sum_a \gamma_a \mathbf{1}(age_i \in a) + \sum_{g,h} \Big(\beta_{g,h} log\left(Sales_i\right) + \gamma_{g,h}\Big)\mathbf{1}(hsn^d(i) = g)\mathbf{1}(hsn^u = h) + \varepsilon_{i,hsn^u} \tag{22}$$

Figure 18: $\psi_{i,hsn}$ controlling for Sales



Note: We plot estimated age fixed effects, γ_a, from Equation 21 along with bootstrap standard errors, normalizing by the youngest age group. The normalized estimates express the difference in $\psi_{i,hsn}$ for firms in a given age category relative to entrants who sell products in the same HSN and have the same sales.

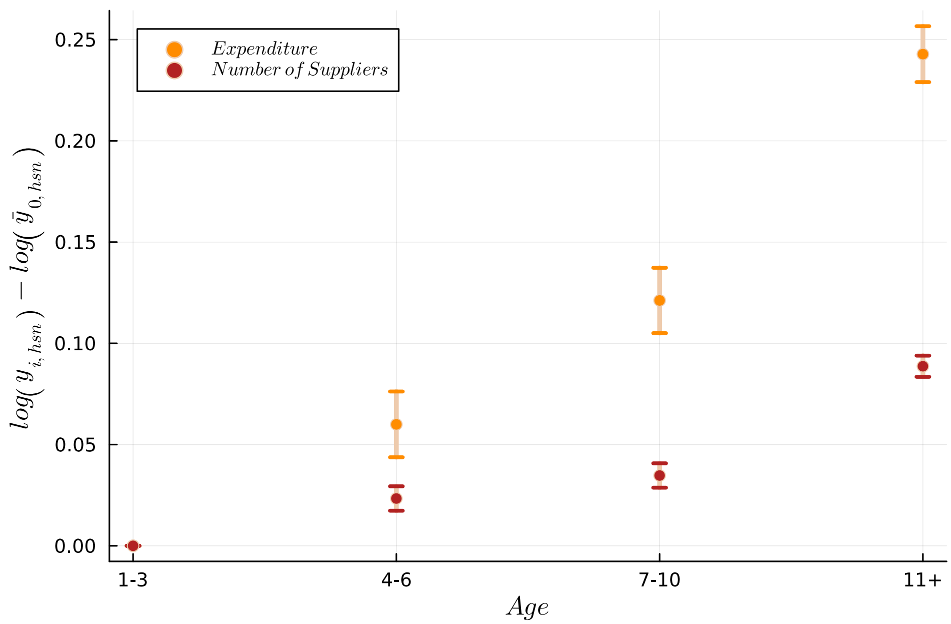
Here, hsn^u refers to a 4-digit HSN from which the firm purchases its inputs, i refers to the firm's Tax ID, and $hsn^d(i)$ refers to the downstream HSN in which the firm sells[7]. $\mathbf{1}(age_i \in a)$ is an indicator variable which equals 1 if the age assigned to the Tax ID is in age category a, $\mathbf{1}(hsn^d(i) = g)$ is an indicator variable which equals 1 if the firm sells its products in the 4-digit HSN category g, and $\mathbf{1}(hsn^u = h)$ is an indicator variable which equals 1 if the firms purchases its inputs from the 4-digit HSN category h.

In Figure 19, we plot lifecycle patterns of number of suppliers and intermediate expenditures, controlling for downstream sales. In particular, we estimate Equation 22 and plot γ_a, normalizing by the youngest age group. The normalized estimates express the difference in log number of suppliers and log intermediate expenditure for firms in a given age category relative to entrants who purchase inputs from the same upstream HSN, sell in the same downstream HSN, and have the same downstream sales. In the case that matching frictions do not affect firm dynamics so that number of suppliers and intermediate expenditures are just a function of firm productivity, age should not matter, after controlling for sales. However Figure 19 shows that, conditional on having the same downstream sales, younger firms

[7] In the data, most firms sell products in multiple HSN categories. We assign $hsn^d(i)$ as the HSN which comprises the greatest share of firm sales. We drop firms for which the greatest HSN category comprises less than 80% of total firm sales.

have fewer suppliers and spend less on intermediate expenditures.

Figure 19: Expenditure and Number of Suppliers controlling for Downstream Sales

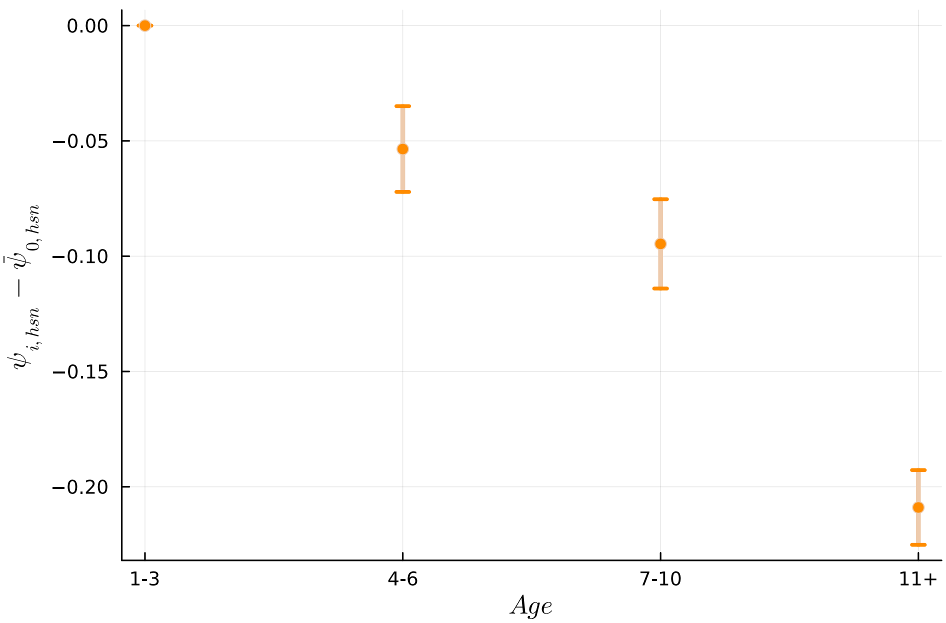


Note: We plot estimated age fixed effects, γ_a, from Equation 22 along with bootstrap standard errors, normalizing by the youngest age group. The normalized estimates express the difference in log number of suppliers (within an HSN) and log intermediate expenditure (within an HSN) for firms in a given age category relative to entrants who purchase inputs from the same upstream HSN, sell to the same downstream HSN, and have the same downstream sales.

In Figure 20, we plot lifecycle patterns of $\phi_{j,hsn}$ from Equation 3, controlling for downstream sales. In particular, we estimate Equation 22 and plot γ_a, normalizing by the youngest age group. The normalized estimates express the difference in $\phi_{j,hsn}$ for firms in a given age category relative to entrants who purchase inputs from the same upstream HSN category, sell in the same downstream HSN, and have the same downstream sales. Noting that $\phi_{j,hsn} = (\sigma - 1) log\,(c_{j,hsn})$, Figure 20 shows that, conditional on having the same downstream sales, younger firms face higher input costs.

Summarizing, controlling for sales, younger firms have fewer customers and lower output prices. Furthermore, controlling for sales, younger firms have fewer suppliers, higher input costs, and lesser intermediate expenditure. These patterns suggest that frictions in firm-to-firm matching play an important role in shaping firm dynamics.

Figure 20: $\phi_{j,hsn}$ controlling for Downstream Sales



Note: We plot estimated age fixed effects, γ_a, from Equation 2 along with bootstrap standard errors, normalizing by the youngest age group. The normalized estimates express the difference in $\phi_{j,hsn}$ for firms in a given age category relative to entrants who purchase inputs from the same HSN, sell in the same downstream HSN, and have the same downstream sales.

B Theoretical Appendix

B.1 Solution Algorithm

In this section, I describe our solution algorithm. A stationary equilibrium is a solution to 6 systems of equations. Equations 23 and 24 are fixed points in payoff-relevant attributes of firms. Payoff-relevant attributes depend on the structure of the network, as seen through $\{H_{z,a}\}$ and $\{G_{z,a}\}$ entering these equations. Equations 25 and 26 are the first order optimality conditions of customer acquisition effort $u(z,a)$ and supplier acquisition effort $v(z,a)$. Optimal acquisition efforts of a (z,a) firm depend on its own payoff-relevant attributes, $c(z,a)$ and $s(z,a)$, and those of other firms, $c(z',a')$ and $s(z',a')$. Equations 27 and 28 are laws of motion for customer and supplier networks implied by the acquisition technology. They depend on the optimal acquisition efforts of firms $u(z,a)$ and $v(z,a)$. I solve for the solution to these systems of equations using a nonlinear solver in Julia.

$$s(z,a) = \left(\frac{\mu c(z,a)^{1-\alpha}}{z\mathcal{P}}\right)^{1-\sigma} X + \int_{Z,A} \left(\frac{\mu c(z,a)^{1-\alpha}}{zc(z',a')}\right)^{1-\sigma} \frac{1-\alpha}{\mu} s(z',a') H_{z,a}(z',a') \tag{23}$$

$$c(z,a)=\left(\int_{Z,A}\left(\frac{\mu c(z',a')^{1-\alpha}}{z'}\right)^{1-\sigma}G_{z,a}(z',a')\right)^{1/(1-\sigma)} \tag{24}$$

$$\frac{\mu-1}{\mu}\sum_{\tau=0}^{\infty}\sum_{z',a'}((1-\delta)\beta)^{\tau}\left(\frac{\mu c(z,a+\tau)^{1-\alpha}}{zc(z',a'+\tau)}\right)^{1-\sigma}\frac{1-\alpha}{\mu}s(z',a'+\tau)\frac{\mathcal{M}(\mathcal{U},\mathcal{V})}{\mathcal{U}}\frac{v(z',a')n(z',a')}{\mathcal{V}}=\frac{\partial\varphi_h(u(z,a))}{\partial u} \tag{25}$$

$$\frac{\mu-1}{\mu}\sum_{\tau=0}^{\infty}\sum_{z',a'}((1-\delta)\beta)^{\tau}\left(\frac{\mu c(z',a'+\tau)^{1-\alpha}}{z'c(z,a+\tau)}\right)^{1-\sigma}(1-\alpha)s(z,a+\tau)\frac{\mathcal{M}(\mathcal{U},\mathcal{V})}{\mathcal{V}}\frac{u(z',a')n(z',a')}{\mathcal{U}}=\frac{\partial\varphi_g(v(z,a))}{\partial v} \tag{26}$$

$$H_{z,a}(z',a')=u(z,a)\frac{\mathcal{M}(\mathcal{U},\mathcal{V})}{\mathcal{U}}\frac{v(z',a')n(z',a')}{\mathcal{V}}+(1-\delta)H_{z,a-1}(z',a'-1) \tag{27}$$

$$G_{z,a}(z',a')=v(z,a)\frac{\mathcal{M}(\mathcal{U},\mathcal{V})}{\mathcal{V}}\frac{u(z',a')n(z',a')}{\mathcal{U}}+(1-\delta)G_{z,a-1}(z',a'-1) \tag{28}$$